Summary: This article is the third of a series intended to summarize our present experience in earth dam design and construction.

Poring in mind the natural heterogeneity of borrow materials, even when accepted as sufficiently homogeneous to frustrate attempts at zoning within the compacted earth section, and further recognizing the inevitable heterogeneity of the compacted product, it is emphasized that the selection of the strength equation for use in stability analyses must be based on statistical reasoning.

A method is proposed for the definition of the strength equation and for subsequent rational presentation of the results of the slope stability so as to permit the conscious selection of a design decision. The procedure, with its implications and consequences, is discussed with references to real examples, including a separation of the design phase and the check testing during construction, and a discussion of the specimen size in comparison with the prototype failure surface.

Sumario: Este artículo es el tercero de una serie destinada a resumir nuestra experiencia en el proyecto y en la construcción de presas de tierra.

Teniendo en cuenta la heterogeneidad natural de los materiales de préstamo, aún cuando son aceptados como suficientemente homogéneos para frustrar tentativas de zonamiento entre las secciones de tierra compactada, y además reconociendo la heterogeneidad inevitable del producto compactado, se acentúa que la selección de la ecuación de resistencia para uso en análisis de estabilidad debe ser basada en razonamientos estadísticos.

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cos. Fué presentado un método para la
definición de la ecuación de resistencia y para la sucesiva
presentación racional de los resultados de la estabili-
dad del talud a fin de permitir la selección consciente de
una decisión de proyecto. El proceso con sus complicacio-
nes y consecuencias, es discutido refiriéndose a ejemplos
verdaderos, incluyendo una separación de la fase del proyecto
y los ensayos de comprobación durante la construcción y una
análisis de las dimensiones del cuerpo de prueba en compara-
ción con la superficie de ruptura del prototipo.

Introduction:-- For those of us who still compute factors of
safety of slopes of dams, and still hope that if present
bases and methods of computations are absolutely wrong, it
might in the long run be more fruitful to search for a
betterment of the situation than discard such attempts by
reliance on experience and judgment, it cannot escape notice
that practically no analysis conducted nowadays seeks to be
realistic.

We shall herein attempt to establish the bases for
a more rational approach for the simplest of cases, that of
the construction period stability of the slope of a
compacted earth dam. It will be assumed that this stability
analysis may rightly be based on the shear strength equation
furnished by Q tests and on the simple statics of the
Swedish-circle.

It should be pointed out that in our experience
the construction period stability seldom proves to be the
determining condition for the design of the slopes of dams
and, therefore, we are not presuming to solve any crucial
problem as regards earth dams. Our intent is to propose an
approach, which can already be applied to this simple case
(as is described herein) and that may doubtless be extended
and adapted, gradually and conscientiously, to apply to the
more complex cases that are really the determining ones.

The only reason why this discussion applies princi-
pally to dams is because their construction is well con-
trolled and therefore the quality imparted to the compacted
product is well known. The reason why it is simple to apply
the proposed approach to the construction period stability
is because the behaviour of the fill in this case may be
considered directly dependent upon the characteristics built
into the fill, and free of indirect influences such as satu-
ration and long-duration effects, etc., on which little is
known and no control can be exercised.

The main points to be considered are (1) how best
to determine by tests, and subsequently compute, representa-
tive shear strength equations for use in the stability com-
putations, and (2) how to present the results of the
stability analyses in a realistic manner.

Heterogeneity of type of soil employed and of compacted product.

It must be conceded that there always is considerable heterogeneity of materials even in a borrow area normally described as uniform. Whether one considers the Atterberg limits of the clayey soil as indicative of its type or whether one uses compaction parameters (Proctor or Harvard Miniature) to distinguish between materials that are not absolutely alike, every borrow area involves a very wide scatter of values indicating changes of material in all directions. Drawing I which represents a small parcel of a remarkably uniform borrow area of the Três Marias Dam (1)* may be taken as an instance.

Furthermore, it is recognized that depending on construction specifications imposed, there will be considerable heterogeneity of the compacted product (2), as is well reflected in the field compaction curves (and corresponding frequency curve of fill water content) shown in Drawing 7.

One must conclude therefore that the selection of a shear-strength equation for use in stability computations must be attempted by statistical reasoning based on the chances of occurrence of the various types of material and of the various qualities of the compacted product of each material.

Depending on the personality of the designer and his feeling of the responsibilities and risks involved, the strength equation has frequently been selected either as the minimum compatible with the strength tests (e.g., tangent to the minimum Mohr circles of failure stresses of the several triaxial compressions run), or as the "average" straight line that by visual inspection seems best to fit all the Mohr circles.

As regards the minimum tangent equation it may be pointed out that since the number of tests is always finite (and frequently not at all large), statistically the minimum and maximum situations are ever yet to occur, if only the number of tests continues to be steadily increased. It is thus seen that there is absolutely no rational basis at all to such a procedure, (unless, again, the minimum be analysed on grounds of probabilities), no matter how well it be presented as representing a worthwhile conservative attitude.

* These numbers correspond to the references listed at the end of the text.
As regards the "average" straight line equation, it would seem fit to establish it by some quantitative procedure, and to establish with reference to the quota of tests available, what degree of assurance is attached to the equation as defined.

Thus, fundamentally both the average and the minimum equations should be established quantitatively on the basis of statistical reasoning. And the principle involved in the choice between average and minimum equations depends basically on the study or evaluation of the relations between the average and minimum strengths of soil elements of the size of the tested specimens, and the corresponding average and minimum cumulative strengths that should prevail along failure surfaces thousands of times bigger in the prototype.

Statistical analysis of triaxial tests on undisturbed specimens from the fill.

The best available method of determining the shear strength of a compacted fill is based on the triaxial testing of specimens carefully cut from undisturbed block samples extracted from the fill at various points. Drawing a summarises the data collected in this manner from 37 triaxial tests on 15 block samples that represent a part of the compacted fill of the Santa Branca dam (1). It is evident that any attempt to establish an average equation visually would be greatly hampered by the considerable scatter, and that the minimum tangent line lies far too low to represent the strength of the fill.

In order to establish statistically the average straight line equation best fitted to represent the shear strength of this compacted fill it is better not to consider tangents to Mohr circles but to substitute each circle by the pair of (σ, τ) values that should represent the stresses on the failure plane, and then to seek the line of best fit passing in between such points.

It seems well agreed (3, 4, 5, 6) that the failure plane in Q tests makes an angle of 45° + 0.5 φ with the plane of the major principal stress, where φ is rather nearly the "angle of internal friction". We therefore begin by establishing the most appropriate value for φ by successive regression analysis using all the available S tests (on the respective compacted fill) run with confining pressures at least 15-20% higher than the "preconsolidation pressure". For the first regression analysis we assume a reasonable value based on the plotted Mohr circles of these high pressure S tests, obtain the (σ, τ) values for the failure planes for each test, and, assuming that these values must fit into an
equation of the type \( T = \sigma \tan \phi \), compute the most probable \( \phi \) value. If this value is not sufficiently close to that assumed, a more appropriate assumption is made for the second cycle regression analysis. This procedure may be repeated until the assumed and computed values almost coincide; it may easily be seen, however, that usually the desired check should be obtained already on the first, or at most on the second cycle.

The \( n \) pairs of values \((\sigma, T)\) representing each Mohr circle of failure stresses in \( Q \) tests may thus be fitted to an equation of the form \( T = \alpha + \sigma \tan \alpha \) by applying the process of the least squares to a linear regression \( y = a + bx \). In drawing 2 the average equation so derived is shown together with tabulated data on various confidence limits of the average computed by use of the standard deviation, the number of tests, and the individual \((\sigma, T)\) values.

**Influence of number of tests and detection of samples not belonging to the same universe.**

Drawings 3 and 4 furnish similar data on undisturbed block samples from zone 1 and 2 of another dam under construction. The zone 1 equations were based on 45 test specimens cut from 13 block samples and those for zone 2 on 22 specimens cut from 9 block samples. These data lead to some conclusions on the detection of construction conditions representing a different statistical universe, and also on the interest of increasing the number of tests.

The zone 1 material indicated considerable scatter so that after performing the initial 12 tests, the program was twice extended, by 6 and 7 additional tests respectively (draw. 4). The first 20 of these tests were run under various chamber pressures, but the final 7 were all run under \( \xi = 5 \) kg/cm². These data were computed at each step for the purpose of furnishing the average equation as well as the lower 90% confidence limit for this whole equation, without considering separate stretches along the variation of \( \sigma \). These results are presented in graph A (draw. 4); the 90% confidence limits appear as a succession of straight lines, as indicated in the graph’s key, because the computation was carried separately for each of the parameters, \( \sigma \) and \( \phi \). It may be noted that the final confidence is naturally greater than 90% if the \( \phi \) and \( \sigma \) parameters be considered absolutely independent, a point on the intersection of the two 90% confidence lines would represent a 99% confidence level.

Graph B (draw. 4) clearly shows how the confidence level rises as the number of tests is increased. One further observes the influence of a concentration of tests with the same chamber pressure. If a straight line equation is sought
for direct use in stability analysis, it appears more useful to concentrate the tests at the upper and lower limits of lateral pressures involved in the specific case. This is always conservative since no strength envelope is known to be concave upward.

Upon attempting to raise even further the confidence limits by performing 10 additional tests for zone 1 and 10 additional tests for zone 2 it was discovered that the new block samples and triaxial tests furnished undisputedly different results (graphs C and D of drawing 4). As a matter of fact these new samples belonged to sections of fill containing somewhat higher average water contents, belonging to truly different statistical universes. The table of Drw. 4 furnishes the results of such a computation demonstrating for zone 1 that the first 27 and last 18 tests furnish equations that are essentially parallel but very significantly different in c-intercept.

Despite the interest in the distinction between the two universes, it is clear that all the tests should be grouped into a single overall universe representing the fill. An attempt was made to compute the standard deviations of unit weights and water contents determined on the several specimens within the various block samples. The average standard deviation of the water content was 1.24%, of the same order of magnitude as within each lot of the fill; that of the unit weight was approximately 1.3 in percent compaction estimated by assuming a unit weight of 1.97 g/cm³ at 100% compaction. Since these deviations are much lower than obtain in the fill, it is clear that several block samples belonging to somewhat different lots will have to be used in order to represent the fill adequately. In view of the importance that we feel should be attached to a true coverage of the range of conditions that obtain in the fill, we are presently using an adapted Hill-Harvard Miniature test to check the percent compaction and water content deviation from optimum for each triaxial specimen.

Before concluding this subject it is worthwhile to comment on the value of special test fills. If the construction equipment is not available so as to reproduce placement and compaction operations under truly representative conditions, not only are such test fills extremely expensive but also the strength results deduced from them may be misleading. Drawing 3 summarizes the test data from two such test fills, and the table in drawing 4 summarizes the statistical tests by which it is proved that only the zone 1 test fill belongs to the same universe as later established in the zone 1 compacted section (possibly because of the wide scatter prevalent), while the zone 2 test fill may not be considered re
Influence of specimen size

An important problem that arises when applying probabilistic reasoning in the search for more realistic expression of the strength of an earth mass, is the problem of extrapolation from small size specimens to large prototypes. It is well known that whatever the type of failure considered, an increase in specimen size results in a decrease of the scatter. Furthermore, it is reasoned that in the theory of brittle failure average values decrease with an increase in size; in the theory of ductile failure, and increase in specimen size may result in slightly smaller, or, in cases, slightly larger, strengths; in the theory of deformation failure, the average strength is not affected by size of specimen. (7)

Drawing 6 furnishes the results of an investigation of the influence of specimen size, under typical conditions of field compaction. Twenty block samples were taken from the Santa Barbara compacted fill, for the purpose of cutting from each block one specimen of each of the diameters, 1.5", 2", 3" and 4", normally used in our laboratories. The material has no grains bigger than 1 mm, with eighty percent of the particles in the silt and clay sizes. Unfortunately the desired program of tests of each size was not fulfilled because of some breakdown during trimming.

The results appear to be of considerable interest since it was determined statistically that the difference between the equations on 1.5", 2", and 3" specimens is significant; the equation on 4" specimens proved unsatisfactory because of insufficient tests. The decrease of the average strength equation with increase in specimen size requires further study, despite its small proportions. It was observed that to some extent this decrease is due to the fact that in trimming the smaller specimens there is a considerably greater proportion of breakage, so that the tests on the smaller specimens exclude the weaker material since only the stronger nuclei survive such operations. It is believed that if this factor is compensated for, the average strength will be found independent of size; the bigger specimens will raise the lower confident limit much in the same way as a greater number of tests.

Stability analyses and safety factors

Although the lower percent confidence lines of shear strength as a function of normal stress are curved; it is an easy matter to transform into an equivalent straight line
equation that part of the curve that should be applicable to
the stability analysis of a given slope. The equivalent
area method is suggested for this transformation (Dwy. J).

Such straight line equations may be used directly
in standard present-day procedures for computing slope stabili-
yty. Drawing 5 presents an example of such an analysis and
the results thereof. It is interesting to observe that the
results will naturally be presented in the form of percent
probabilities of the computed factor of safety dropping to
a given value (and the most reasonable value to adopt would
be unity). The stability analysis presented does not yet
embody fully the concept of probabilities since only the con-
fidence limits on the strength equation were included and
none of the other factors of variation or error, either in
assumptions or in procedures, were considered. The result
is submitted merely in support of the proposal of a method
of reasoning that is believed to embody the principal hope
of development of our profession. We are apologists of the
fact that no matter how one look at the engineering profes-
sion, every decision to build something requires on the part
of the owners a choice that amounts intrinsically to a
gamble - at present a highly disguised gamble with very
small odds of failure usually accepted, but nevertheless, an
inevitable gamble. It is therefore believed proper and
necessary to present the results in such a manner that one
may consciously select one's odds - in contradiction to
present procedure wherein one has no way of discovering what
they are.

A further interest in such an approach derives from
the fact that it is time possible to estimate the value of
additional and more representative testing. If the confi-
dence limits rise and thereby a steepening of slopes is ac-
cepted it is possible to evaluate how far the reduction in
volume economically justifies additional investigations.

Preliminary investigations in design stage

Present-day laboratory test programs for design are
usually based on the selection of one or two "representative
borrow materials; on each a set of 6 tests is run on ap-
methods compacted to some selected average condition, such as
"93% compaction" and "optimum minus 1/2" water content.

Our present procedure involves the following steps.
The materials occurring in a borrow area are first sub-
divided into groups (e.g. the shaded area of Dwy. 1): within
each group it is assumed that all materials, each molded to
the same percent compaction and water content deviation from
optimum, would yield approximately the same strength para-
meters. A rationalization of such a selection by groups is being attempted by further study of the influence of the type of soil on its properties as compacted, at similar compaction parameters. (8).

Thereupon the molding characteristics of the specimens should be established on the basis of estimated field compaction data. In dw. 7 we reproduce, from the data of a dam under construction, the roller compaction curve with its confidence limits and the corresponding frequency distribution curve of compaction water content (the confidence limits on this curve may be neglected because of the great number of tests). From such a combination of curves it is easy to see how to schedule a program of tests for representative molding characteristics; two examples are furnished in the tables accompanying this drawing.

One must emphasize the importance we attach to the recognition of the true range of parameters that are likely to occur in the field on the basis of a given set of construction specifications (2). In view of the difficulty in fully anticipating this range, we recommend that design computations normally aim at slightly higher factors of safety so as to avoid placing the base of the fill narrower than may later be proved desirable. Separate test fills are not considered recommendable. During the initial placement and compaction under normal full-scale construction operations, careful field control (1,2) and testing of undisturbed block samples should promptly furnish data for any revision of the design, if desired.

Conclusions

Shear strength equations for use in stability analyses may be established quantitatively, despite natural scatter, by use of statistical computations, both for average conditions and for various confidence limits of such an average. The stability of a slope may also be represented more appropriately with respect to probabilities of an undesirable situation developing. All testing should be carefully programmed and checked to represent the true range of variations expected or encountered in the fill; the influence of the number of tests on the confidence limits is appreciable, and that of the specimen sizes requires consideration.

Conclusions

Las ecuaciones de resistencia al corte para uso en las analíses de estabilidad pueden ser establecidas cantitativamente, a despecho de la natural dispersión, con el
empleo de cálculos estadísticos, no solo para condiciones médicas como para varios límites de confianza de la media. La estabilidad de un talud puede también ser representada más apropiadamente con referencia a las probabilidades del desenvolvimiento de una situación no deseada. Todos los ensayos deben ser cuidadosamente programados y confrontados para representar el intervalo real de variaciones esperadas o encontradas en el relleno; la influencia del número de ensayos en los límites de confianza es apreciable, y la de las dimensiones de las muestras requiere consideración.

REFERENCES


INFLUENCE OF NUMBER OF TESTS AND DETECTION OF SAMPLES BELONGING TO DIFFERENT UNIVERSE

ZONE 1 - Q TESTS

ZONE 2 - Q TESTS

DRW-DIBUJO 4