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# *8th de Mello Lecture*

# *Role Played by Viscosity on the Undrained Behavior of Normally Consolidated Clays*

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# *Role Played by Viscosity on the Undrained Behavior of Normally Consolidated Clays*

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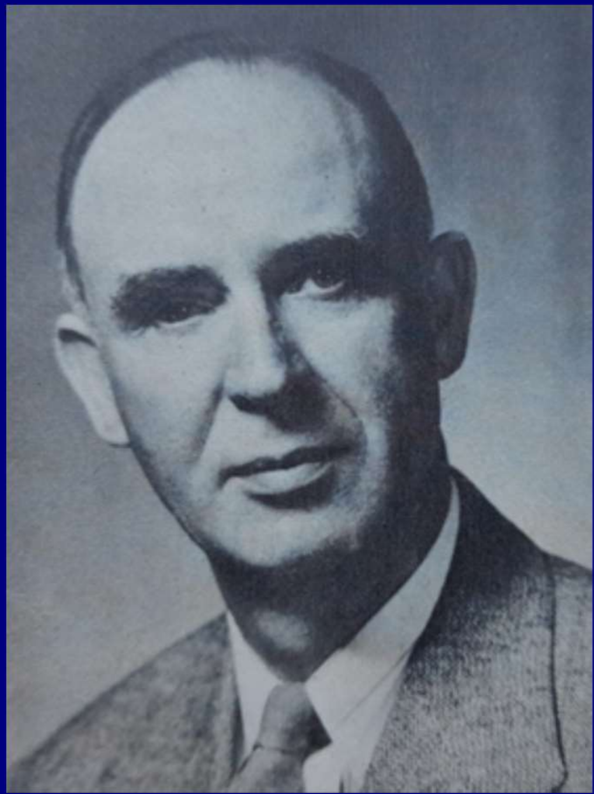


*Prof. Donald Wood Taylor*

*Partnership between Donald Wood Taylor and Victor de Mello extended up to the last year of Taylor's life in 1955, six years after Victor de Mello had left MIT to live in Brazil.*

## ***The Principle of Effective Stress (PES)***

***PES fundamental equation*** $\Rightarrow$ $\sigma' = \sigma - u$

***Change in the state of effective stress*** $\Leftrightarrow$ ***Change of volume or distortion (or both)***

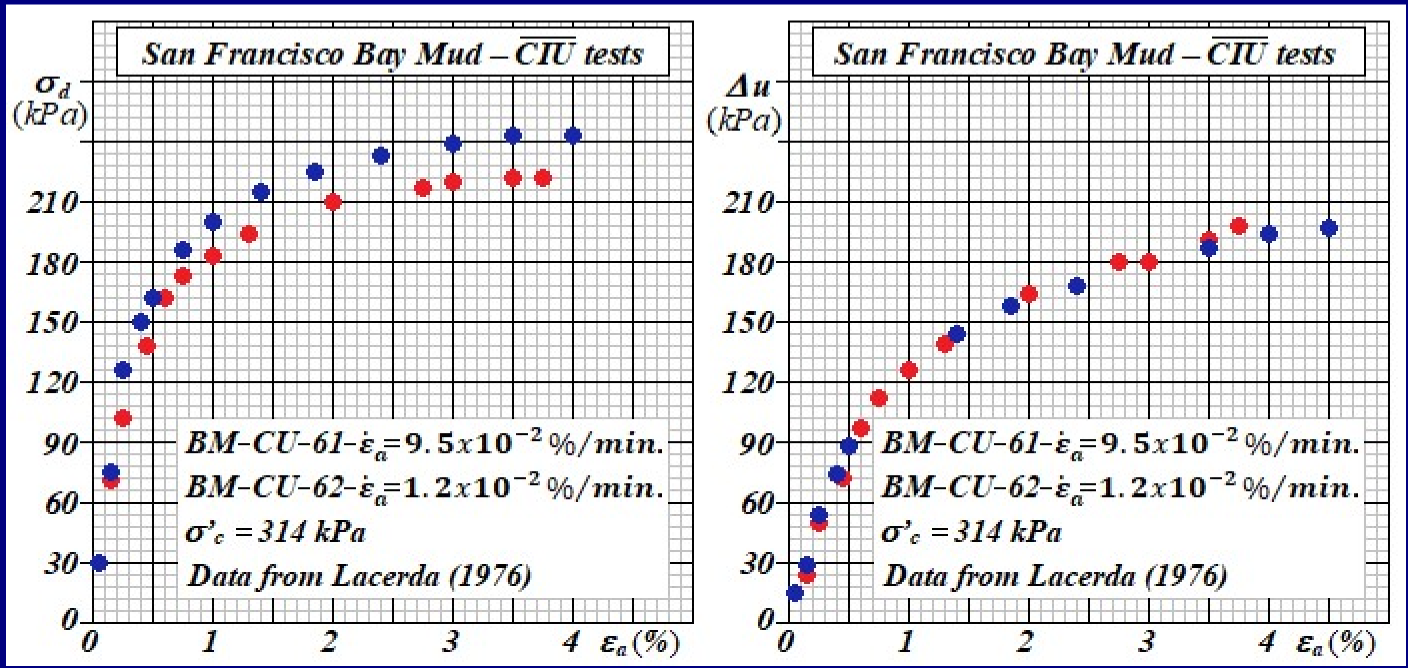
*Atkinson e Bransby (1978) present 3 corollaries of the* ***PES - 2 of which are given below***

*Corollary 1: The engineering behaviour of two soils with the same structure and mineralogy will be the same if they have the same effective stress.*

*Corollary 2: If a soil is loaded or unloaded without any change of volume and without any distortion there will be no change of effective stress.*

The corollaries presented above illustrate how the ***PES*** is understood and used by soil mechanics practitioneers.

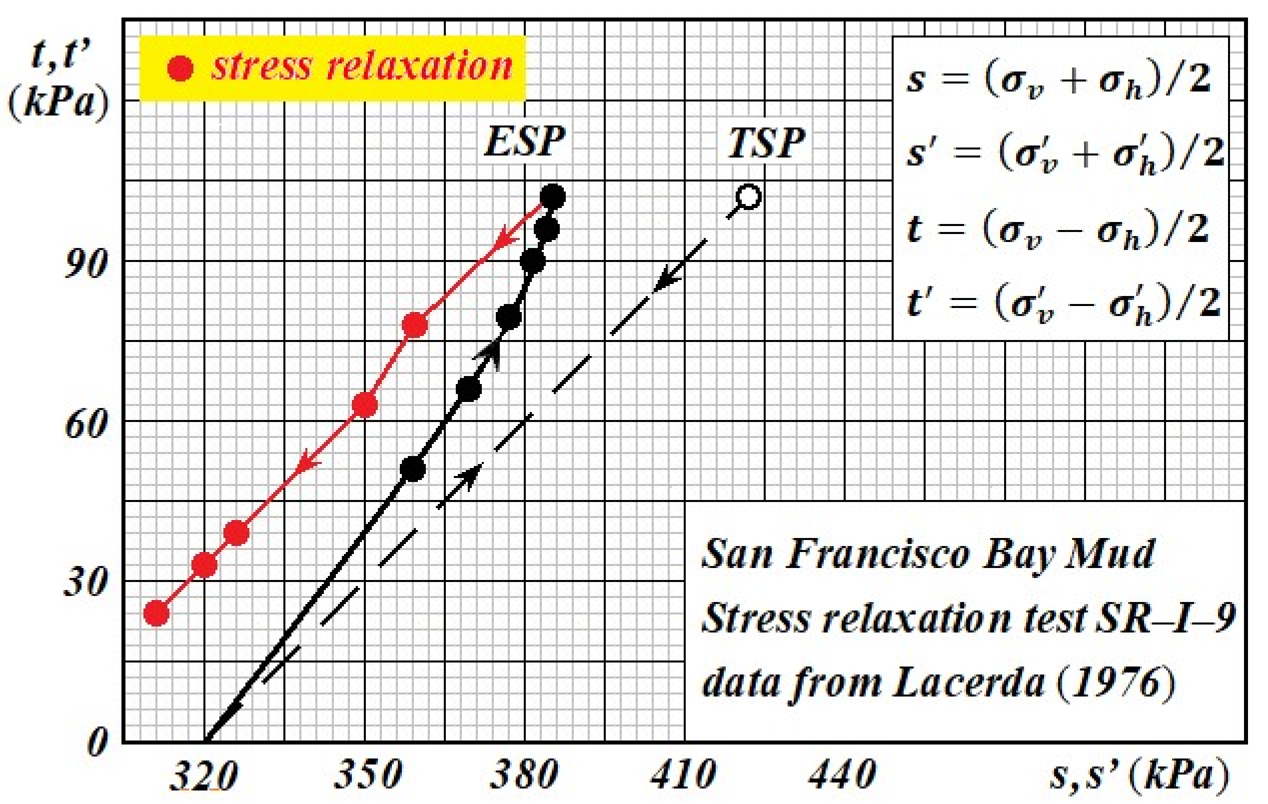
Example of soil behaviour that does not obey corollary 1




Two equal specimens of the same soil (thus with the same structure and mineralogy) departing from the same state of effective stress, but tested with different axial strain rates ($\dot{\varepsilon}_a$) present different behaviour.

Strain rate does affect undrained strength but does not affect pore pressure generation.

Example of soil behaviour that does not obey corollary 2




- $\overline{CIU}$ conventional triaxial test during which the load frame motor is switched off . A stress relaxation follows (decrease of deviator stress with time).

- In a $\overline{CIU}$ test on a saturated soil with the load frame motor switched off , there is neither volume change nor distortion. However, there is a change in effective stress.

- These examples show there are cases where the ***PES*** does not hold true.

- The examples showing the ***PES*** is not of general validity are usually found among phenomena involving strain rate effects and time effects like creep and stress relaxation.

- In view of the theoretical difficulties of dealing with such phenomena, the usual attitude is to preserve the essence of the ***PES*** and to develop an additional tool to tackle the specific problem which is outside the ***PES*** framework. An example is the $C_\alpha/C_c$ = constant approach, proposed by Mesri and Godlewski (1977), to evaluate secondary consolidation.

- This lecture follows an attitude different from the usual.

- The ***PES*** is enlarged to encompass phenomena that are outside its domain of validity, like strain rate effects, undrained creep and stress relaxation, making them natural consequences of a more general ***PES***.

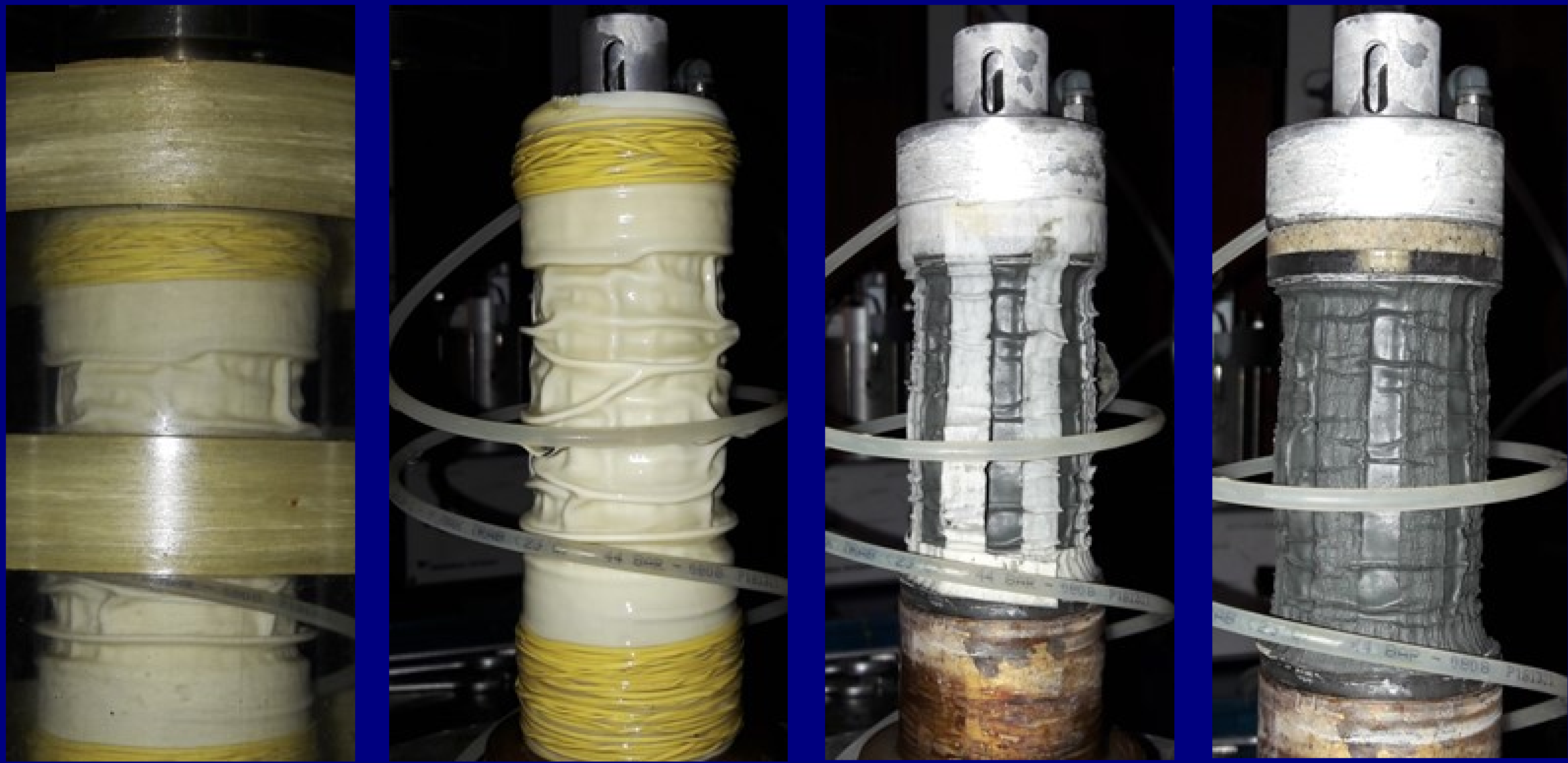
- The concepts that allow this ***PES*** enlargement are already available in classic texts like Terzaghi and Frölich (1936), Terzaghi (1941), Taylor (1942), Taylor (1948), Hvorslev (1960), Bjerrum (1973) and Leroueil et al. (1985), illustrating one of de Mello's thoughts: "*We professionals beg less rapid novelties, more renewed reviewing of what is already there*".

- Main aim of this lecture $\longrightarrow$ to study the relationship between causes and effects of strain rate as it affects the undrained behaviour of plastic soils.

- This study will be limited to isotropic normally consolidated saturated clays, without cementation between grains, subjected to undrained axisymmetric states of stress like those found in triaxial tests of the $\overline{CIUCL}$ (***Consolidated Isotropically Undrained Compression Loading***) type.

- One of the effects of axial strain rate ($\dot{\varepsilon}_a$) is to increase the undrained strength ($S_u$) of clayey soils, defined by $S_u = \left(\sigma'_{af} - \sigma'_{rf}\right)/2$, where $\sigma'_{af}$ and $\sigma'_{rf}$ are respectively the effective axial and radial stresses at failure in $\overline{CIUCL}$ triaxial tests.

Axial strain rate effect on the undrained strength of Sarapuí River Clay measured in a $\overline{\boldsymbol{CIUCL}}$ test carried out with lubricated ends (“free ends”).



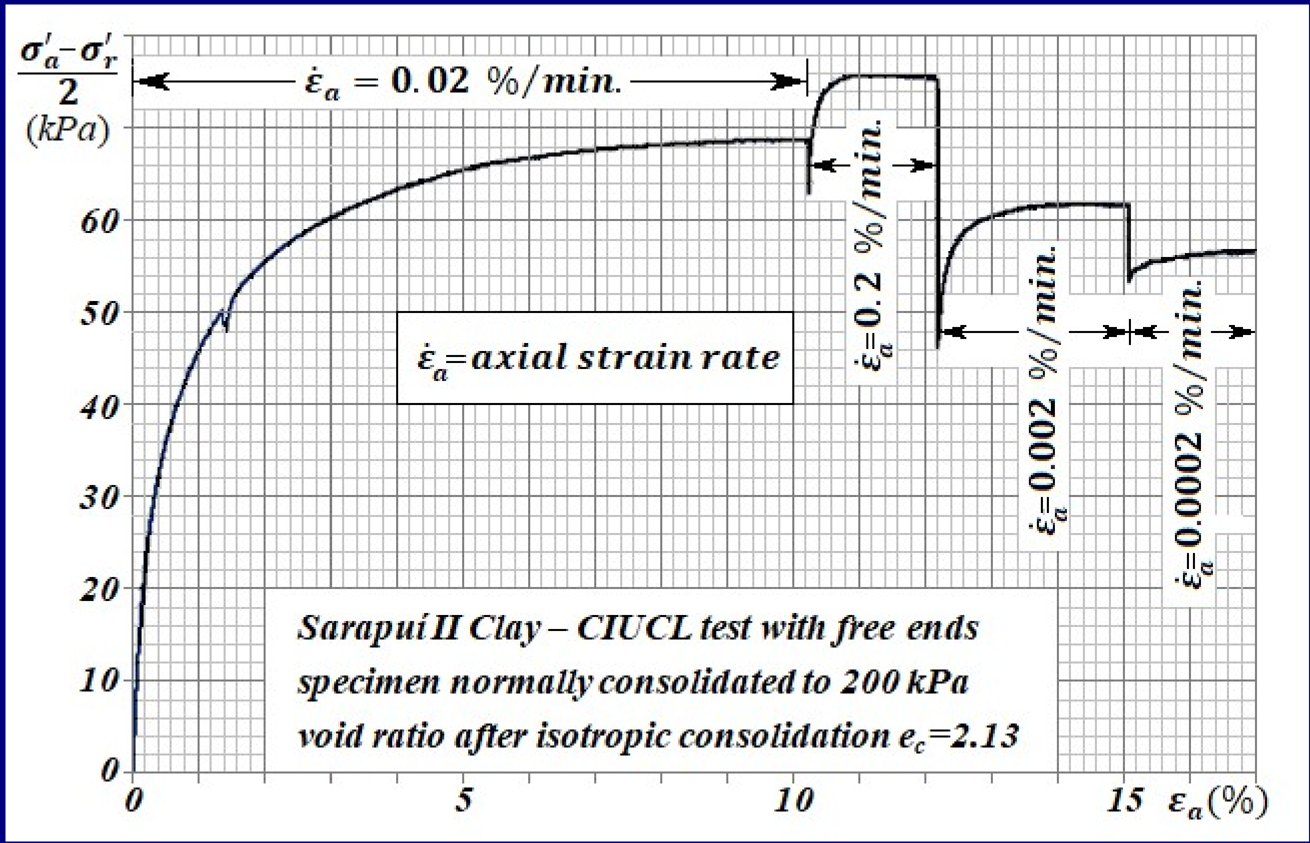
(a) Inside the cell chamber at the end of test with $\varepsilon_a$=17%.

(b) At the end of test outside the cell chamber with the rubber membrane.

(c) Without rubber membrane but with filter paper.

(d) Without rubber membrane and filter paper.

Typical result showing the influence of strain rate on the undrained strength ($S_u$) of the highly plastic (***P I> 100 %***) Sarapuí River Clay II measured in a $\overline{CIUCL}$ triaxial test (with free ends).

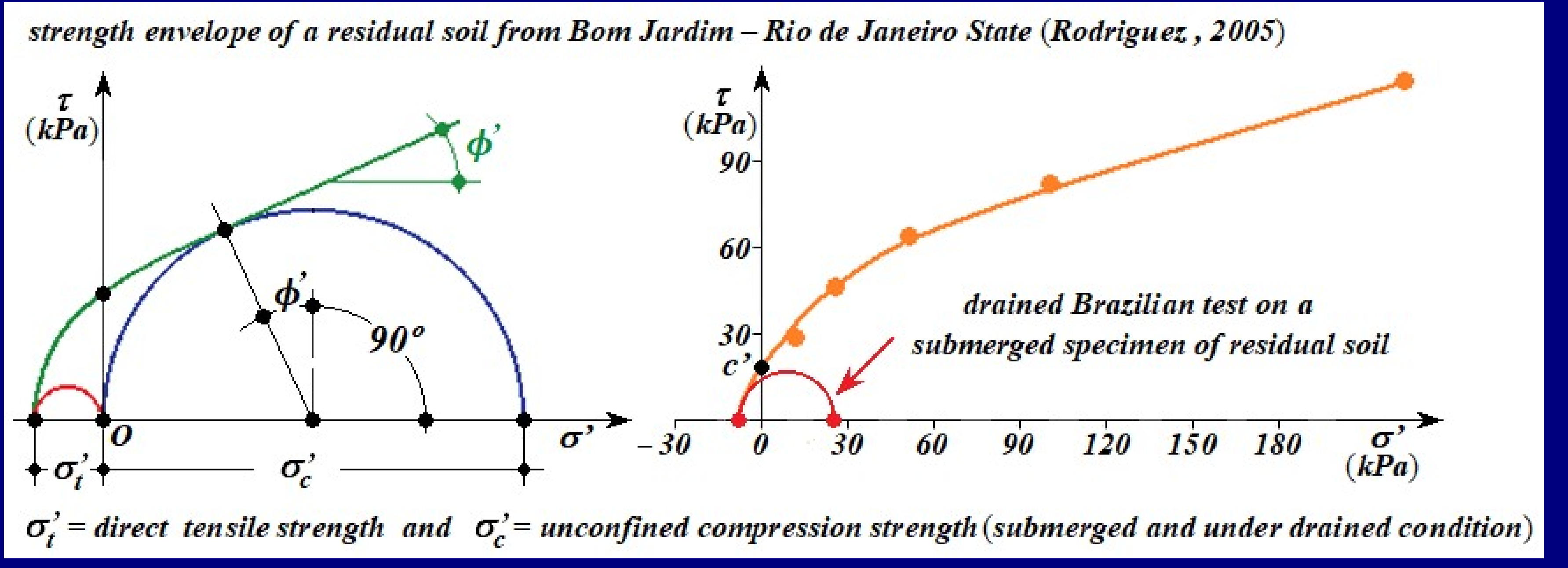


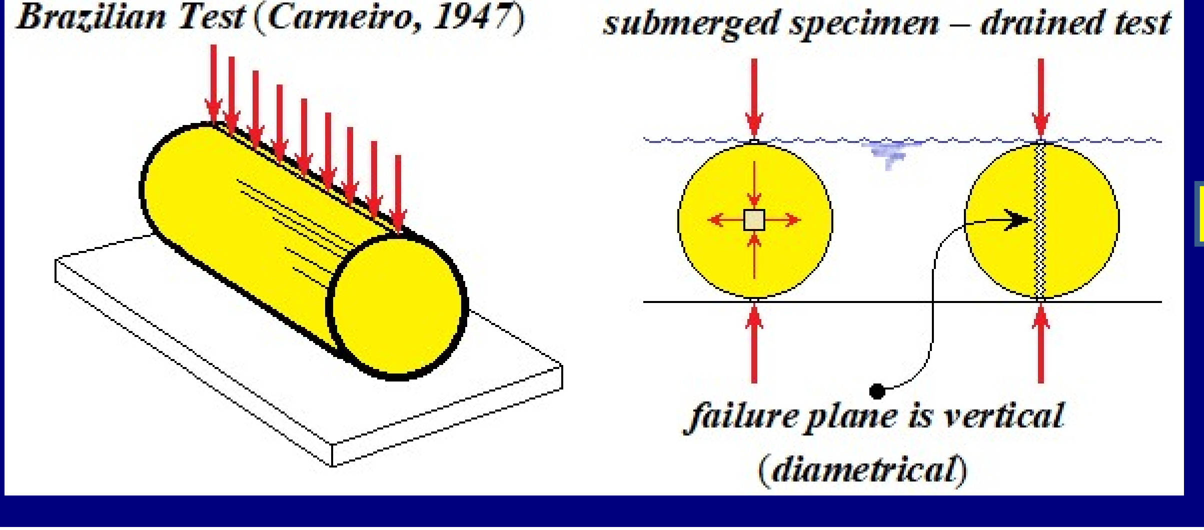
Bjerrum (1973) erimental evidences show that strain rate effects are associated to the “cohesive component” of shear strength as it was defined by Hvorslev (1960).

However, as posed by Schofield (1999,2001) and assumed in critical state soil mechanics, soils do not have cohesion in the sense used by Coulomb. How to solve this disagreement ? After all, what is cohesion ? Do soils have cohesion or do not ?

**Strength envelope of a residual soil**

*True cohesion as defined by Coulomb (can be assigned to cementation between grains remaining from the mother rock during weathering).*

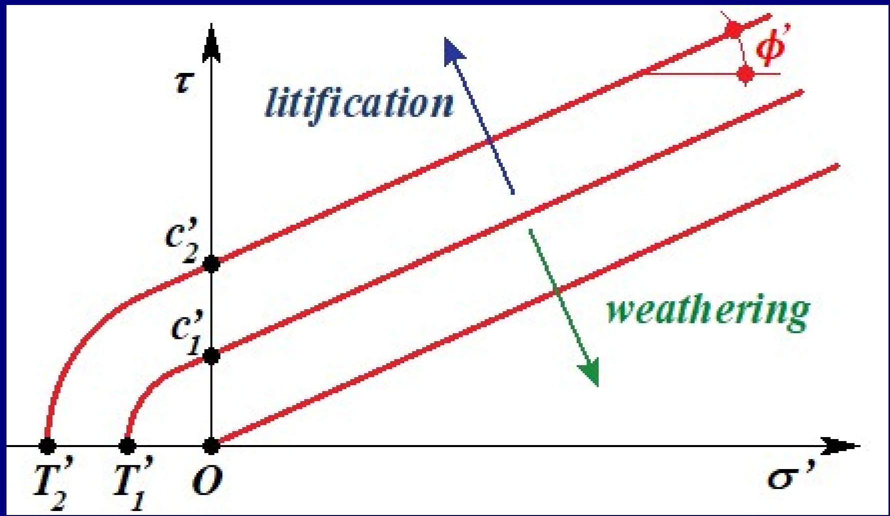



***Tensile strength determined by Brazilian test on a submerged specimen under drained condition.***

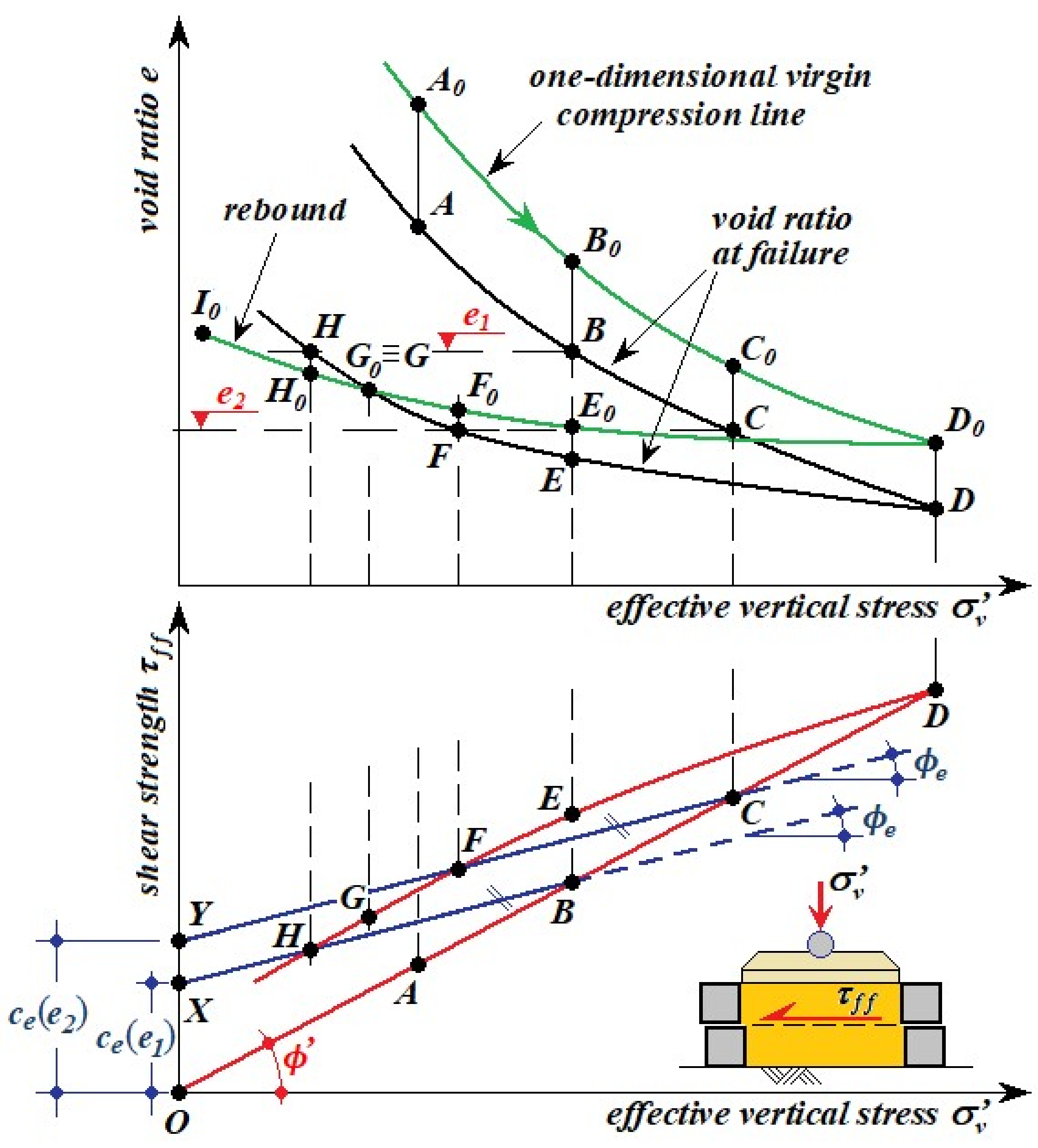
True cohesion , weathering and litification

Weathering ➡ natural process by which rocks are transformed in residual soils. Weathering causes loss of true cohesion with time (loss of cementation between grains existing in the mother rock).




In the inverse process, called diagenesis, sedimentary soils are transformed in sedimentary rocks. During the process of litification there is a gain of true cohesion due to the cementation between soil grains (see effect on the strength envelope in figure above).

On the Hvorslev true cohesion, cohesive soils, plasticity and viscosity.




(1) Drained direct shear tests on a saturated remoulded clay.

Hvorslev (1937), Terzaghi (1938), Gibson (1953).

(2) ***OABCD*** ➡ Strength envelope – normally consolidated condition.

(3) A normally consolidated clay does not have true cohesion in the physical sense defined by Coulomb.

(4)***DEFGH*** ➡ overconsolidated strength envelopes depend on $\sigma'_{ff}$ (normal effective stress on the failure plane at failure, equal to $\sigma'_v$) and void ratio or water content at failure).

Hvorslev (1937) observed and concluded that:

(1) Points representing the shear strength $\boldsymbol{\tau_{ff}}$ of saturated remoulded clay specimens with the same void ratio (or water content) at failure fall on a straight line whose equation is $\boldsymbol{\tau_{ff} = c_e(e) + \sigma'_{ff}\, tan\, \phi_e}$.

(2) The $\boldsymbol{\tau_{ff}}$ intercept $(\boldsymbol{c_e})$ of the straigth line envelope is a function of the void ratio at failure. The slope $\boldsymbol{tan\, \phi_e}$ is the same for all straight line envelopes, e.g. all points on the straight line envelope ***XHB,*** whose $\boldsymbol{\tau_{ff}}$ intercept is $\boldsymbol{c_e(e_1)}$, have void ratio at failure equal to $\boldsymbol{e_1}$.

(3) $\boldsymbol{\tau_{ff}}$ intercept $\boldsymbol{c_e}$ is a function of the void ratio (or water content) and the slope $\boldsymbol{tan\, \phi_e}$ is a constant.

(4) The shear strength of a clayey soil is expressed by: $\boldsymbol{\tau_{ff} = c_e(e) + \sigma'_{ff}\, tan\, \phi_e}$

(5) $\boldsymbol{c_e}$ was called the "***true cohesion***" (*sic*) being a sole function of the void ratio at failure.

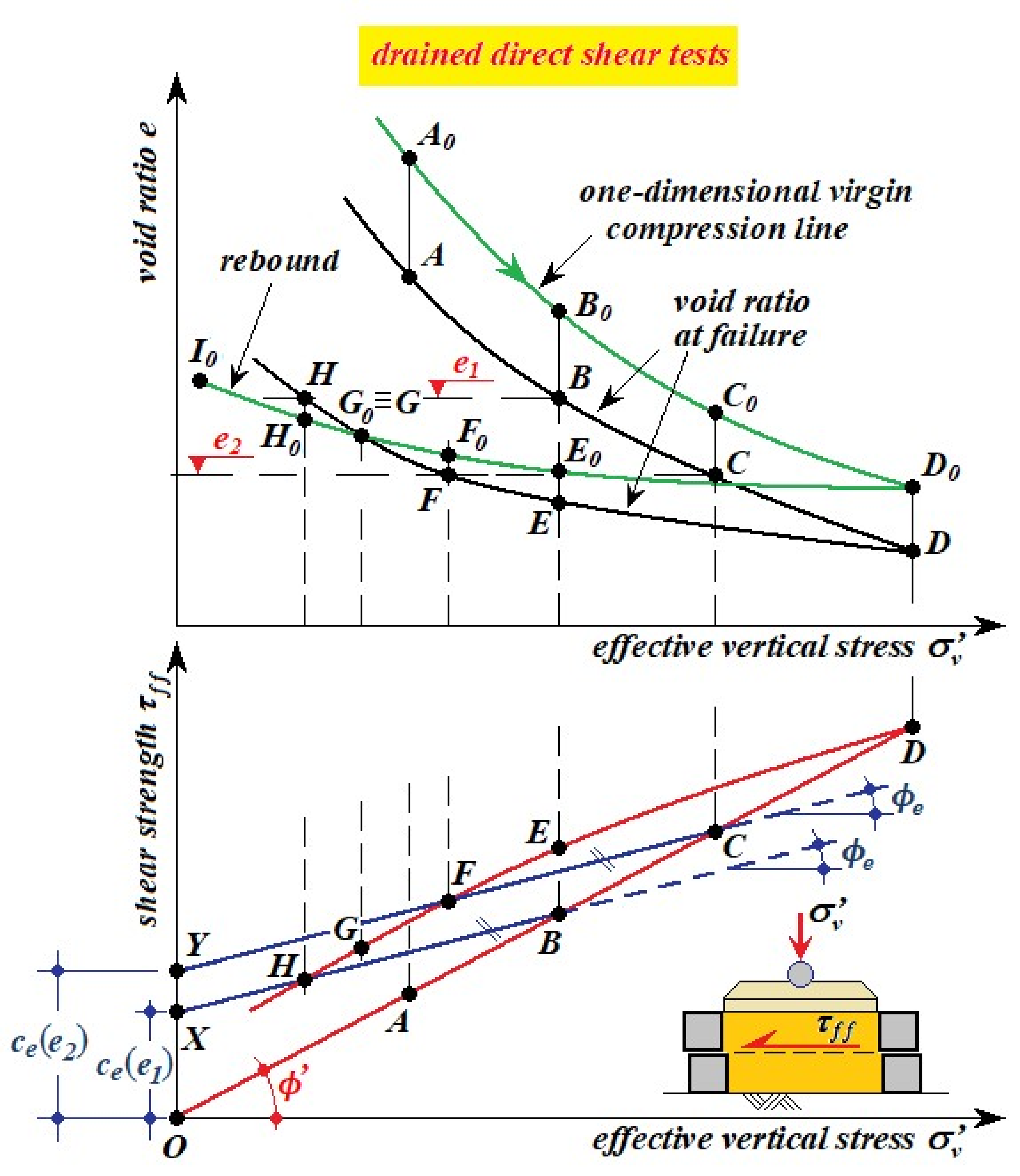
$\boldsymbol{\sigma'_{ff}}$ is the normal effective stress on the failure plane at failure.

$\boldsymbol{\phi_e}$ was defined as the ***"true angle of friction".***

The word "***true cohesion***" used by Terzaghi (1938) and Hvorslev (1937, 1960), expressed as a sole function of the void ratio (or water content) at failure, has a different physical meaning from the same word used by Coulomb in spite of both having the dimension of a stress.

Question: As the clays tested by Hvorslev(1937) were remoulded, they could not present cementation among the grains and thus, according to the Coulomb cohesion concept, the use of the term ***"true cohesion"*** would not be appropriate.

Then, what would be the correct physical meaning of the term $\boldsymbol{c_e}$ in the expression $\boldsymbol{\tau_{ff} = c_e(e) + \sigma'_{ff}\, tan\, \phi_e}$ ?

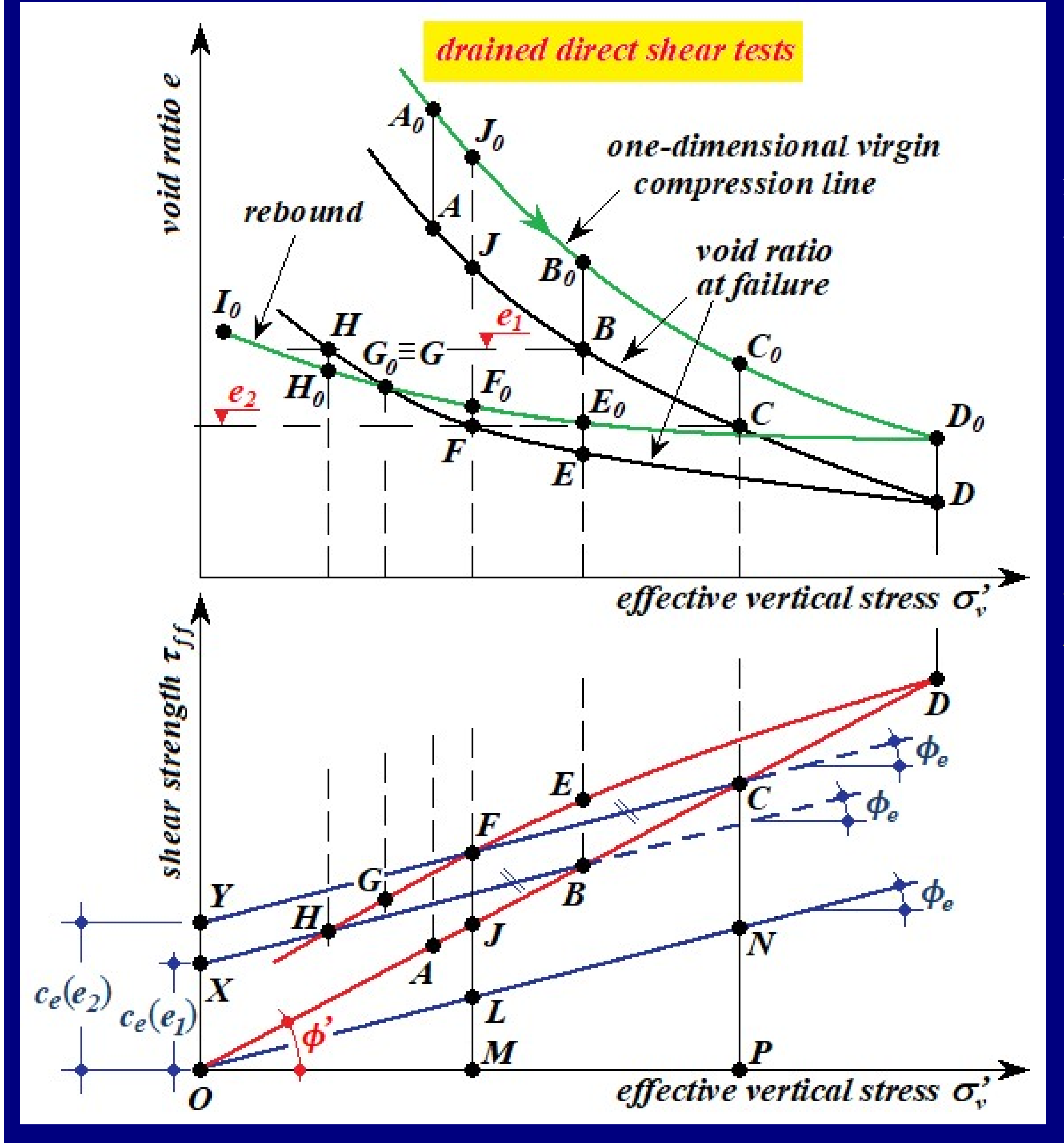



(1) On the line ***HB*** the void ratio at failure is $\boldsymbol{e_1}$. Hence, on the $\boldsymbol{\tau_{ff}}\ x\ \boldsymbol{\sigma'_v}$ plot, points ***H*** and ***B*** define a straight line envelope with slope $\boldsymbol{\phi_e}$, for which the Hvorslev "***true cohesion***" is a constant equal to $\boldsymbol{OX} = \boldsymbol{c_e}(e_1)$

(2) Along the straight envelope ***XHB*** there is only variation of the frictional component given by $\boldsymbol{\sigma'_v}\ \boldsymbol{tan}\ \boldsymbol{\phi_e}$.

(3) In a similar way, along the line ***YFC*** there is only variation of the frictional component. In this case, the void ratio $\boldsymbol{e_2} < \boldsymbol{e_1}$. The Hvorslev ***"true cohesion"*** is a constant equal to $\boldsymbol{OY} = \boldsymbol{c_e}(e_2)$.

Hvorslev (1937) strength envelope x Mohr-Coulomb (normally consolidated) envelope.




(1) The straight line ***OAJBCD*** is the strength envelope corresponding to the normally consolidated (virgin) domain where the shear stress at failure $\boldsymbol{\tau_{ff} = \sigma'_{ff}\, tan\, \phi'}$

(2) The same envelope can be written as $\boldsymbol{\tau_{ff} = c_e(e) + \sigma'_{ff}\, tan\, \phi_e}$. But in this case $\boldsymbol{c_e(e)}$ varies along ***OAJBCD*** because the void ratio is varying !

(3) Subtracting the ordinates of the ***OLN*** straight line from those of the ***OAJBCD*** straight line, one obtains a linear function of $\boldsymbol{\sigma'_{ff}}$ (or $\boldsymbol{\sigma'_v}$), i.e.:

$$\boldsymbol{c_e(e) = \sigma'_{ff}\, tan\, \phi' - \sigma'_{ff}\, tan\, \phi_e = C.\sigma'_{ff}}$$

Thus, the shear strength $\boldsymbol{\tau_{ff}}$ of a normally consolidated clay can be written as

$$\boldsymbol{\tau_{ff} = \sigma'_{ff}\, tan\, \phi' = c_e(e) + \sigma'_{ff}\, tan\, \phi_e = C.\sigma'_{ff} + \sigma'_{ff}\, tan\, \phi_e}$$

Dividing both members of the above equation by $\boldsymbol{\sigma'_{ff}}$,

$\boldsymbol{tan\, \phi' = C + tan\, \phi_e}$ , with $\boldsymbol{C = constant}$

A portion of the shear strength of a normally consolidated clay, which does not possess cohesion in the sense used by Coulomb, is due to a resistance that cannot be assigned to friction and that was improperly called "cohesion" by Terzaghi (1938).

Anyway what is behind the expression "***effective cohesion***" or ***"true cohesion"*** as used by Hvorslev and Terzaghi in an inadequate way ?

# THÉORIE DU TASSEMENT
DES
# COUCHES ARGILEUSES

## INTRODUCTION A LA MÉCANIQUE ANALYTIQUE DES ARGILES

PAR

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*Traduit de l'allemand par*

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DES PONTS ET CHAUSSÉES

PARIS
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92, RUE BONAPARTE (VI)
1939

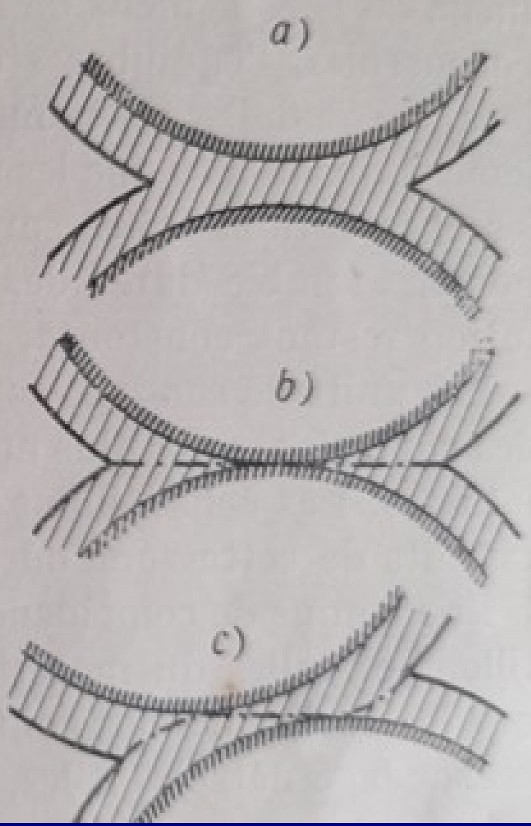
Terzaghi and Frolich (1936) – Translated from German to French by  Marc Adler (1939)



passe parfois des années et des dizaines d'années avant que les grains ne se touchent (fig. 7*b*). Dans une couche d'argile qui reste pendant des milliers d'années sous l'influence de son poids propre constant, ce contact devient inévitable. La résistance au glissement des particules n'est pas produite uniquement par la résistance de frottement mais aussi par la viscosité des couches séparatrices, entourant la zone de frottement.

Dans cet état, l'argile doit forcément présenter les propriétés élastiques d'un amas de grains dont les points de contact sont reliés rigidement les uns aux autres.

Après le glissement (fig. 7 *c*), les particules sont de nouveau séparées par une couche de fluide visqueux. La mobilité des particules augmente avec l'épaisseur de cette couche; le coefficient de compressibilité, par contre diminue.



It seems clear that the Terzaghi (1938) ***"cohesion"*** and the Hvorslev (1937,1960) ***"effective cohesion"*** or ***"true cohesion"*** have a viscous origin … and viscous resistance depends on the speed of shear !

In the author's opinion is the viscous nature of the "***cohesion"*** which is behind the correction factor $\mu$ that should be applied to the results of a vane test, (which are higher than that observed in failure cases in the field, due to the fact of being obtained using a high speed of shear).

Bjerrum (1973) expressed the correction factor $\mu$ as a function of the plasticity index. The higher the plasticity index, the higher the expected ***"cohesion"***, i.e. the viscous resistance, effect of the adsorbed water.

Quoting from Hvorslev (1960)

*"Most cohesive soils possess an apparent structural viscosity and their deformations are of visco-elastic character. The corresponding strength component may be called the "viscous component", but factors other than viscosity seem to be involved, and the more inclusive term "rheological component" and the notation* $\boldsymbol{c_v}$ *are proposed. It will be assumed that* $\boldsymbol{c_v}$ *forms a part of the effective cohesion component,*$\boldsymbol{c_e}$ *...*

*For the purpose of definition and experimental determination of the individual components* (individual components of shear strength)*, the basic assumption is made that the cohesion and rheological components* (read viscous resistance) *are constant when*

***(1) the void ratio or water content of saturated clays is constant.***

***(2) the rate of deformation or test duration is constant.***

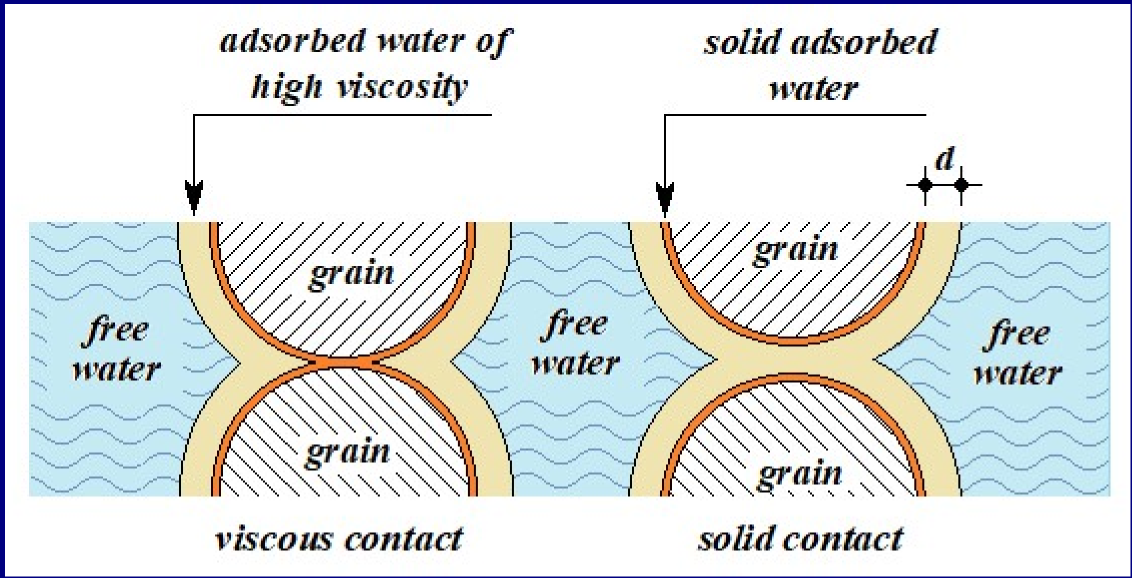
***(3) there is no significant difference in the geometric structure of the clays during a given test series.***

## *Quoting from Taylor ( 1948 ) – Fundamentals of Soil Mechanics ( pp. 379-380 )*

*"The effect of speed of shear on the strength is believed to be caused by the viscous or plastic characteristics of material in the adsorption zones in the vicinity of points of contact or near contact of clay particles. Thus, this effect is a colloidal phenomenon, and it is of sufficient importance to justify a detailed discussion."*

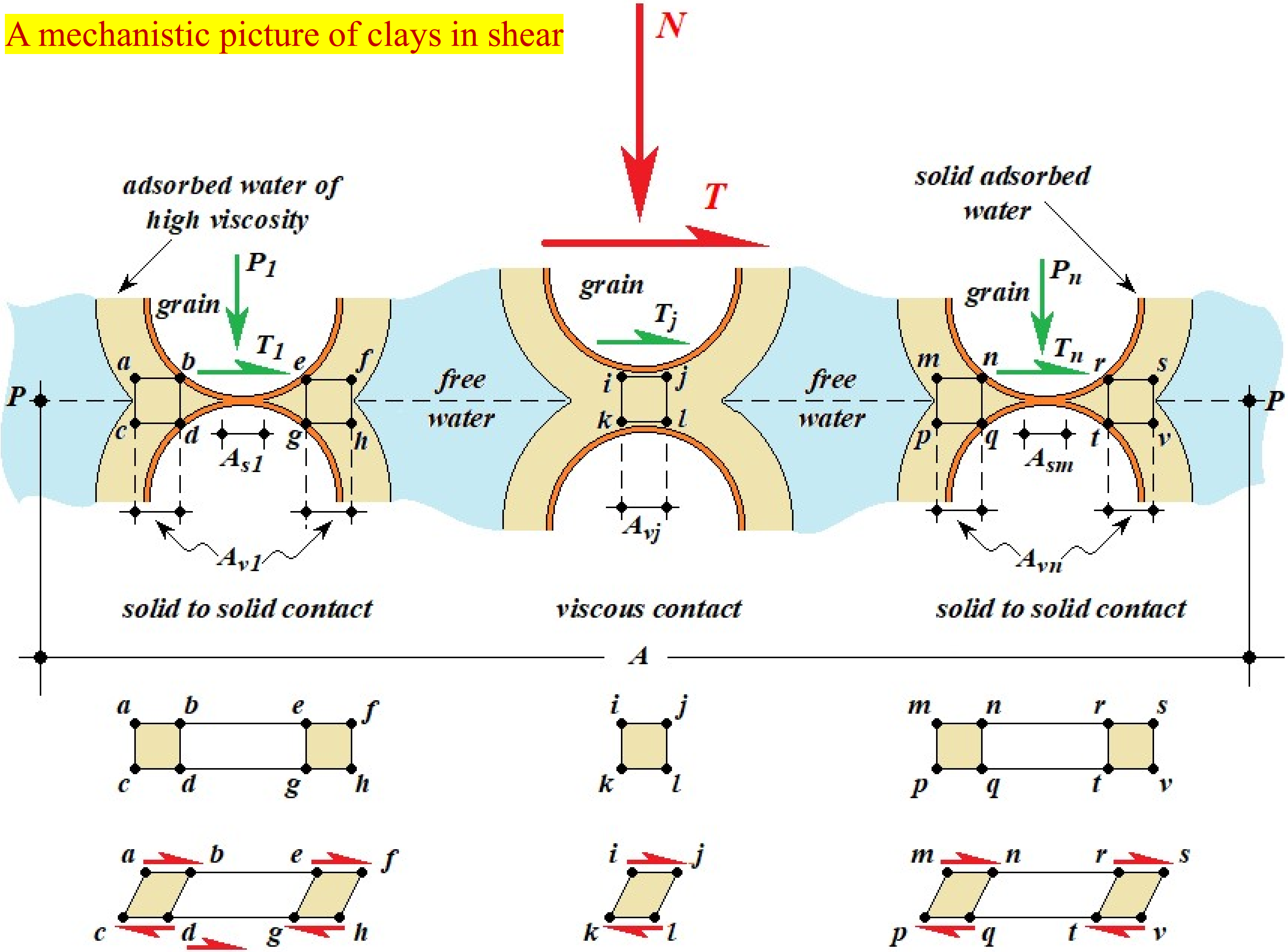
*"The following hypothetical explanation of plastic resistance and of time relationships was first presented (Taylor,1942) for one-dimensional compressions, but it may be extended to the action of clays in shear. If a drained clay sample is maintained under any given system of constant applied direct and shearing stresses that do not cause failure, it gradually approaches an ultimate shape and an ultimate void ratio at which there is static equilibrium. Ages may be required to reach this state of equilibrium, but when it is reached the applied stresses are equal to static internal resistances and they have values that are free of plastic resistance and all other time effects. During the approach to equilibrium, however, the applied stresses are made up in part of the stresses required to overcome the plastic resistance. The plastic resistance is usually considered to depend mainly on the speed of strain although possibly it depends also on such factors as changes in type or degree of adsorption. As the clay specimen approaches the static case, the strains continuously decrease in speed and the plastic resistance decreases in magnitude; however, the speed become almost imperceptibly small when the plastic resistance is still quite large and the strains and the void ratio still have a considerable change to undergo before they reach the static case. Secondary compression, as it occurs in consolidation tests, is a good illustration of this condition. From these concepts it appears that a clay that has reached static equilibrium in nature after the lapse of many centuries and is suddenly subjected to stress increases of relatively small magnitude may be expect immediately to exert a plastic resistance that is equal to the stress increase, and it is possible that the speed of distortion required for the exerting of this amount of plastic resistance may be too small to be noticeable. In such a case the plastic resistance cannot be distinguished from a bond, and the occurrence of bonds of this type is possible both when the shearing stresses are small and when they are relatively large".*

Terzaghi (1941) Beyond the thickness where adsorbed water is solid, the water is highly viscous, the viscosity decreasing with distance from the particle surface.




For distances greater than a limit value “d”, viscous water becomes free water. The thickness “d” depends on the chemical properties of the solid particles and substances within the adsorption zone, e.g. sodium salts.

A mechanistic picture of clays in shear



Equilibrium along plane $\boldsymbol{P}-\boldsymbol{P}$ assuming adsorbed water as a newtonian liquid gives

$$\boldsymbol{\tau = \tau_\phi + \tau_\eta = \sigma' \tan \phi'_{mob} + \eta(e)(d\gamma/dt)}$$

Conclusion: a shear stress $\boldsymbol{\tau}$ acting on a plane of a plastic soil mass is resisted internally by the sum of two components: a frictional component $\boldsymbol{\tau_\phi = \sigma' \tan \phi'_{mob}}$ and a viscous component $\boldsymbol{\tau_\eta = \eta(e)(d\gamma/dt)}$.

The above equation translates mathematically how Terzaghi and Frölich (1936), Hvorslev (1937, 1960) and especially Taylor (1948) conceived the behaviour of clays during shear.

**In plastic soils part of the shear resistance, improperly called "*cohesion*", is of viscous nature and depends on the shear strain rate $(\boldsymbol{d\gamma/dt})$.. When the shear strain rate becomes zero, viscous resistance vanishes. This behaviour does not correspond to the Coulomb concept of cohesion but to the viscosity concept as presented by Newton.**

- The viscous portion $(\boldsymbol{\tau_\eta})$ of shear stress corresponds to that portion of shear strength Hvorslev (1937,1960) and Terzaghi (1938) improperly called "true cohesion".

- If shear stresses are internally resisted by a frictional component and a viscous component, one can add one more equation to the ***PES.***

- Such equation is ⇨ $\boldsymbol{\tau = \tau_\phi + \tau_\eta = \sigma' \, tan \, \phi'_{mob} + \eta(e)(d\gamma/dt)}$.

- When $\boldsymbol{\sigma' = 0}$, friction resistance becomes zero and the above equation leads to Newton's law of viscosity. So, the whole equation holds true even when the water content is above the (physical) liquid limit (fluid mechanics domain).

- Bjerrum (1973) captured the essence of the phenomenon and established a relationship between strain rate effects and plasticity index ( ***PI*** ). The higher the ***PI***, the greater the influence of the strain rate on the shear strength of a plastic soil.

Finally, to make the confusion of using improperly the term ***“cohesion”*** come to an end, instead of using the expression ***“cohesive soil”***, it is suggested to replace it by the expression ***“plastic soil”***.

From a practical point of view and to avoid misunderstanding, every soil which presents liquid and plastic limits is meant to be a plastic soil.

The Mohr circle, the viscosity ellipse and the friction ellipse.

Considering $\boldsymbol{\tau_\eta = \eta(e)(d\gamma/dt)}$ and assuming $\boldsymbol{\eta(e)}$ is the same irrespective of the plane considered (soil is isotropic) and remembering that distortion $\boldsymbol{\gamma_\alpha = 2\varepsilon_{s\alpha}}$, the viscous component $(\boldsymbol{\tau_{\eta\alpha}})$ of the shear stress along a plane that makes an angle $\boldsymbol{\alpha}$ with the direction of $\boldsymbol{\varepsilon_1}$ is given by

$$\boldsymbol{\tau_{\eta\alpha} = \eta(e)(d\gamma_\alpha/dt) = \eta(e)(d2\varepsilon_{s\alpha}/dt) = 2\eta(e)(d\varepsilon_{s\alpha}/dt)}$$

With $\boldsymbol{\varepsilon_{s\alpha}} = [(\boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_3)/\mathbf{2}]\ \boldsymbol{sin\ 2\alpha}$

In a given instant of a triaxial test, irrespective of it being drained or undrained, the ***state of mobilized viscosity*** is given by:

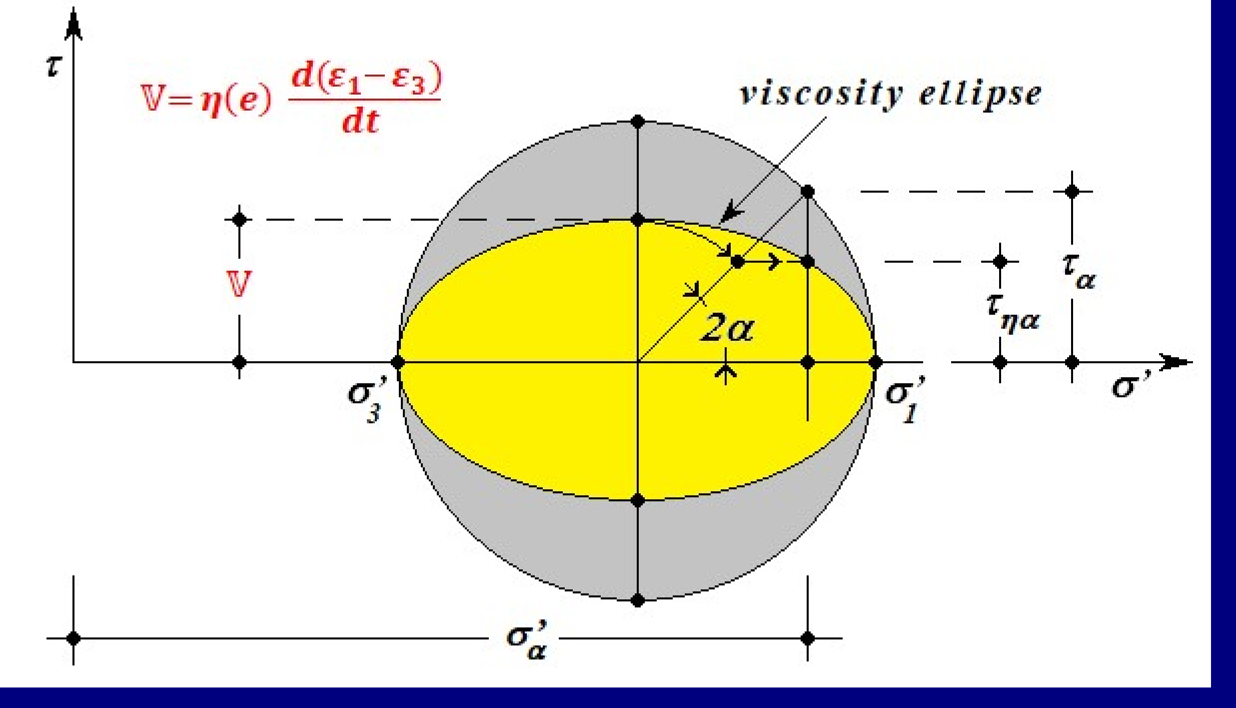
*State of mobilized viscosity*

$$\sigma'_\alpha = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\alpha$$

and

$$\tau_{\eta\alpha} = \eta(e)\ \frac{d(\varepsilon_1 - \varepsilon_3)}{dt} \sin 2\alpha$$

These are the parametric equations of an ellipse of centre at $[(\sigma'_1 + \sigma'_3)/2\,,0]$, major axis equal to $(\sigma'_1 - \sigma'_3)$ and minor axis equal to $\eta(e)\,d(\varepsilon_1 - \varepsilon_3)/dt$ - ***the Viscosity Ellipse.***



The maximum ordinate of the viscosity ellipse is denoted by $\mathbb{V}$ and given by:

$$\mathbb{V} = \boldsymbol{\eta(e)}\,\frac{\boldsymbol{d(\varepsilon_1 - \varepsilon_3)}}{\boldsymbol{dt}} = \boldsymbol{\eta(e)\,\dot{\gamma}}$$

The friction component $\boldsymbol{\tau_{\phi\alpha}}$ acting on the same plane where $\boldsymbol{\tau_{\eta\alpha}}$ acts is given by:

$$\boldsymbol{\tau_{\phi\alpha} = \tau - \tau_{\eta\alpha}} = \left[\frac{\boldsymbol{(\sigma_1' - \sigma_3')}}{\boldsymbol{2}} - \mathbb{V}\right] \boldsymbol{sin\,2\alpha}$$

***State of mobilized friction***

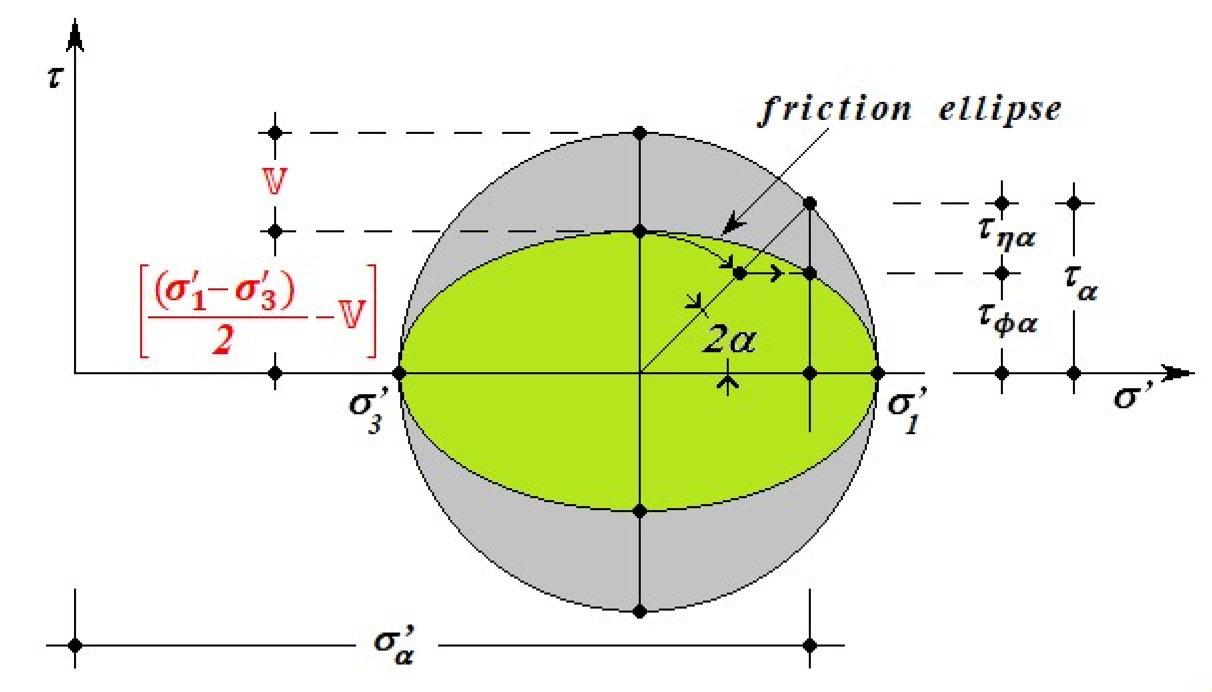
$$\boldsymbol{\sigma_\alpha'} = \frac{\boldsymbol{\sigma_1' + \sigma_3'}}{\boldsymbol{2}} + \left(\frac{\boldsymbol{\sigma_1' - \sigma_3'}}{\boldsymbol{2}}\right) \boldsymbol{cos\,2\alpha}$$

and

$$\boldsymbol{\tau_{\phi\alpha} = \tau - \tau_{\eta\alpha}} = \left[\frac{\boldsymbol{(\sigma_1' - \sigma_3')}}{\boldsymbol{2}} - \mathbb{V}\right] \boldsymbol{sin\,2\alpha}$$

These are the parametric equations of another ellipse of centre at $[\boldsymbol{(\sigma_1' + \sigma_3')/2\,,0}]$, major axis equal to $\boldsymbol{(\sigma_1' - \sigma_3')}$ and minor axis equal to $[\boldsymbol{(\sigma_1' - \sigma_3') - 2}\mathbb{V}]$. ***The Friction Ellipse*.**

*The friction ellipse*




- Mohr circle of effective stress = friction ellipse + viscosity ellipse.

- Both ellipses cannot exist alone because only the Mohr circle fulfills static equilibrium.

- Shear stress $\tau_\alpha$ acting on a plane whose normal makes an angle $\alpha$ with $\sigma_1$ direction is made up of two components: a friction component $\tau_{\phi\alpha}$ and a viscous component $\tau_{\eta\alpha}$.

A failure criterion for plastic soils taking into account the strain rate

This presentation will be restricted to $\overline{\boldsymbol{CIUCL}}$ (Consolidated Isotropically Undrained Compression Loading) tests.

As $\boldsymbol{\varepsilon_v = \varepsilon_a + 2\varepsilon_r = o \rightarrow \varepsilon_r = -(\varepsilon_a/2)}$

During the undrained shearing of a conventional $\overline{\boldsymbol{CIUCL}}$ test ⇨ $\boldsymbol{d\varepsilon_a/dt = cte}$.

Conclusion: The viscous component of the shear stress acting on a fixed plane $\alpha$ of a conventional $\overline{\boldsymbol{CIUCL}}$ test specimen is constant during all the test.

⇩

$$\boldsymbol{\tau_{\eta\alpha} = \eta(e)\,\frac{d(\varepsilon_1 - \varepsilon_3)}{dt}\,sin\ 2\alpha = \frac{3}{2}\eta(e)\frac{d\varepsilon_a}{dt}\ sin\ 2\alpha}$$

**Conclusions of the assumed working hipotheses:**

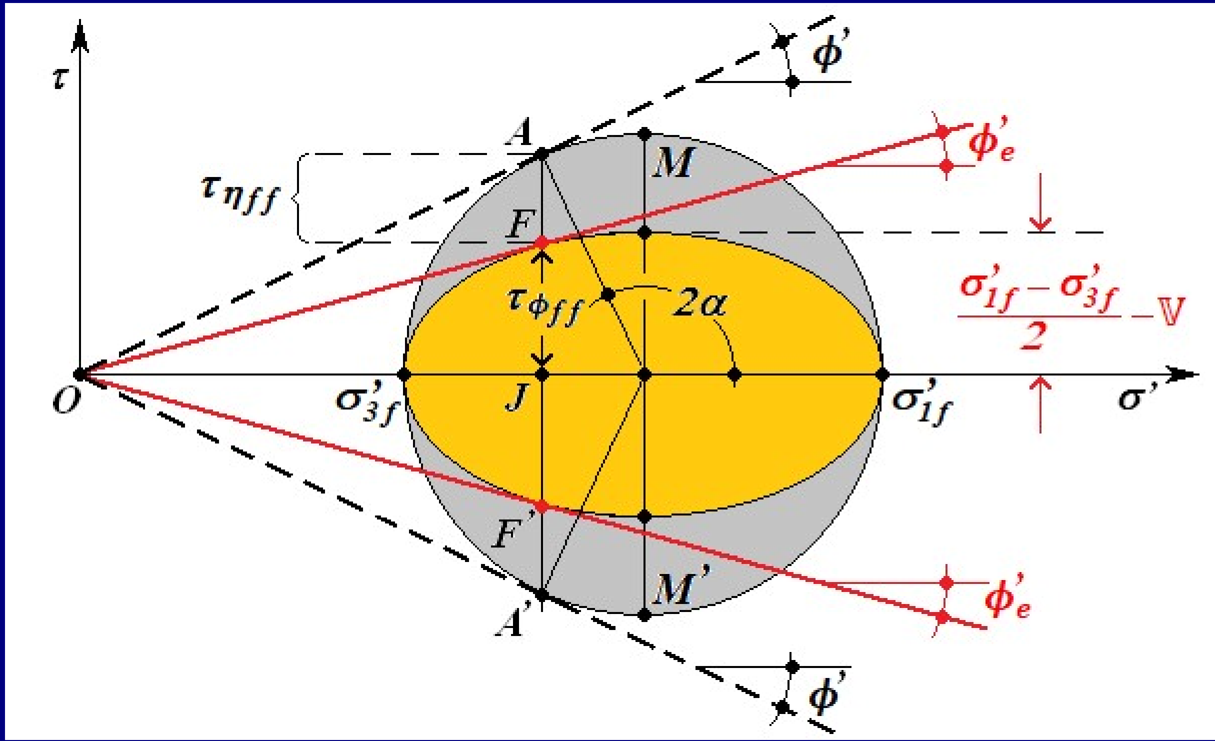
- During the shearing phase of a $\overline{CIUCL}$ test with axial strain rate $\dot{\varepsilon}_a = cte.$, the viscous component is instantaneously and fully mobilized as soon as the load frame motor is switched on. Then, the viscous component is kept constant in each and every plane of the specimen until the end of the test.

- The deviator stress increases as the specimen is deformed with $\dot{\varepsilon}_a = constant$ until failure. As the viscous component is fully mobilized since the very beginning of the test, one comes to the conclusion that the increase of the deviator stress with strain is due to the friction component mobilization.

- As in the shearing stage of a $\overline{CIUCL}$ $\varepsilon_v = 0$, during undrained shear there are only distortions ➡ conclusion: friction mobilization is associated to shear strains.

- Failure occurs when the frictional resistance is fully mobilized. Hence, shear deformation and failure are governed by friction mobilization.

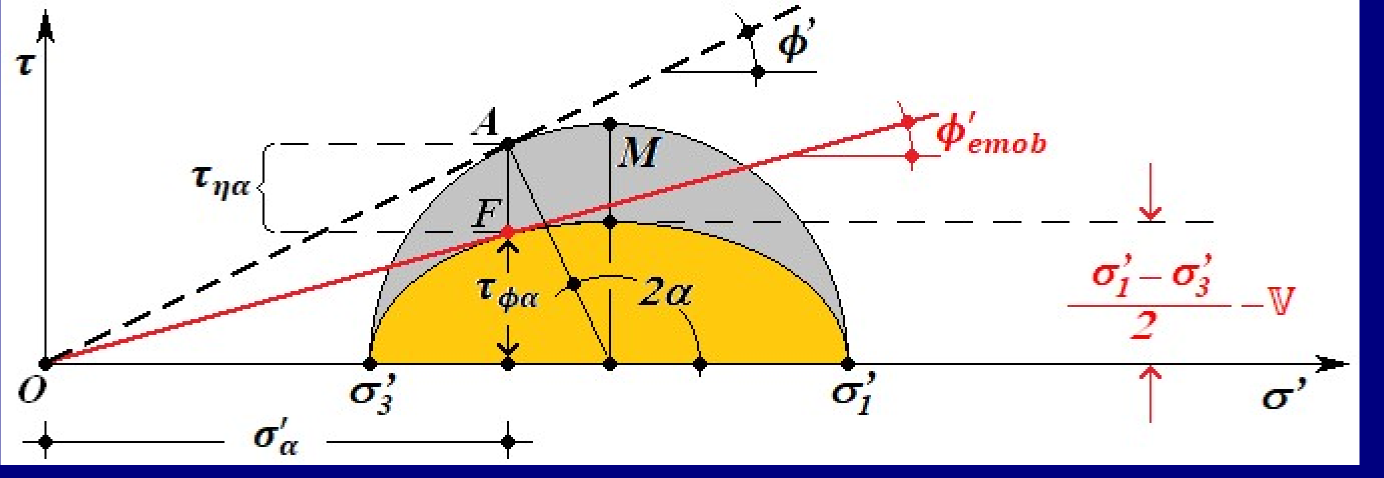
Failure occurs when the friction component is fully mobilized, i.e. when the friction ellipse becomes tangent to the failure envelope. For normally consolidated clays the failure envelope is the straight line through the origin of the plane $\tau \ x \ \sigma'$ with slope $\boldsymbol{tan\ \phi'_e}$, being $\boldsymbol{\phi'_e}$ the Hvorslev true angle of internal friction.




$$\boldsymbol{\tau_{ff} = \tau_{\phi ff} + \tau_{\eta ff}}$$

- $\boldsymbol{\tau_{ff}}$ = ***shear stress on the failure plane at failure.***
- $\boldsymbol{\tau_{\phi ff}}$ = ***friction component of*** $\boldsymbol{\tau_{ff}}$.
- $\boldsymbol{\tau_{\eta ff}}$ = ***viscous component of*** $\boldsymbol{\tau_{ff}}$ .

- $\phi'_{emob}$ = maximum angle of obliquity.
- At each and every instant of the test ⇒ $\tan\phi'_{emob} = \left(\tau_{\phi\alpha}/\sigma'_{\alpha}\right)_{max}$
- $\phi'_e$ maximum angle of obliquity at failure. $\tan\phi'_e$ ⇒ maximum value of $\tan\phi'_{emob}$




Maximum angle of obliquity occurs on a plane where ⇒ $\tan\phi'_{emob} = \left(\tau_{\phi\alpha}/\sigma'_{\alpha}\right)_{max} \rightarrow \dfrac{\partial\left({}^{\tau_{\phi\alpha}}/{}_{\sigma'_{\alpha}}\right)}{\partial\alpha} = 0$

$$\frac{\partial\left({}^{\tau_{\phi\alpha}}/{}_{\sigma'_{\alpha}}\right)}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left\{\frac{\left[\frac{(\sigma'_1-\sigma'_3)}{2} - \mathbb{V}\right]\sin 2\alpha}{\frac{(\sigma'_1+\sigma'_3)}{2} + \frac{(\sigma'_1-\sigma'_3)}{2}\cos 2\alpha}\right\} = \frac{\partial}{\partial\alpha}\left[\frac{(t'-\mathbb{V})\sin 2\alpha}{s' + t'\cos 2\alpha}\right] = 0$$

$$\frac{\partial\left(\tau_{\phi\alpha}/\sigma'_{\alpha}\right)}{\partial\alpha} = \frac{2(t'-\mathbb{V})\cos 2\alpha.(s'+t'\cos 2\alpha)+2(t'-\mathbb{V})\sin 2\alpha.t'\sin 2\alpha}{(s'+t'\cos 2\alpha)^2} = 0$$

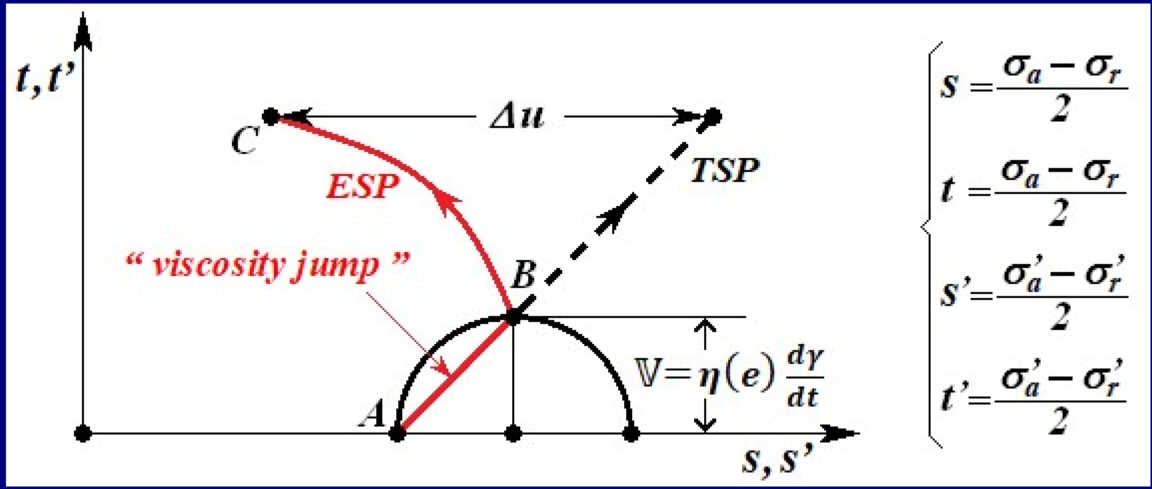
Solving for $\boldsymbol{\alpha}$ gives $\boldsymbol{\cos 2\alpha = -\frac{\sigma'_1-\sigma'_3}{\sigma'_1+\sigma'_3}}$, thus

$$\tan \phi'_{emob} = \frac{\left(\frac{\sigma'_1-\sigma'_3}{2}-\mathbb{V}\right)}{\sqrt{\sigma'_1\sigma'_3}} = \frac{(t'-\mathbb{V})}{\sqrt{s'^2-t'^2}}$$

At failure $\boldsymbol{\tan \phi'_{emob} = \tan \phi'_e}$ and hence

$$\tan \phi'_e = \frac{\left(\frac{\sigma'_{1f}-\sigma'_{3f}}{2}-\mathbb{V}\right)}{\sqrt{\sigma'_{1f}\sigma'_{3f}}} = \frac{(t'_f-\mathbb{V})}{\sqrt{{s'_f}^2-{t'_f}^2}}$$

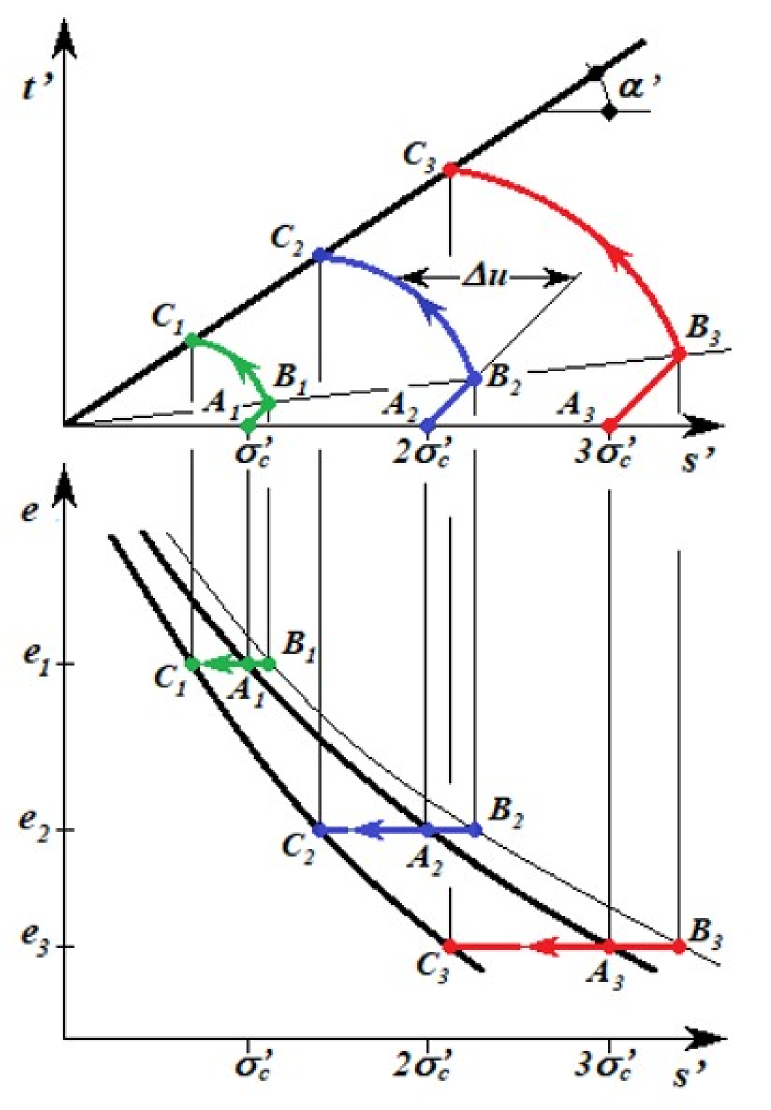
The "***viscosity jump***" in the effective and total stress paths




"***viscosity jump***"
$t' = \mathbb{V} = \eta(e)\,\dot{\gamma}$

- ***AB*** ➡ ***"viscosity jump"*** occurs immediately after the load frame motor is switched on
- Along ***AB*** ➡ total and effective stress paths are coincident.
- ***"viscosity jump"*** causes the effective stress path (***ESP***) of a normally consolidated clay to go from ***A*** to ***B***. Then the ***ESP*** changes its direction at ***B*** and moves to the left towards ***C***.
- At ***B*** there is only viscous resistance mobilized (mobilized friction resistance is zero).

Effective stress paths on the planes $\boldsymbol{s'}\,x\,\boldsymbol{t'}$ and $\boldsymbol{e}\,x\,\boldsymbol{s'}$ for a given $\dot{\boldsymbol{\varepsilon}}_t = constant$.




$$\boldsymbol{s'} = \frac{\boldsymbol{\sigma'_1 + \sigma'_3}}{\boldsymbol{2}}$$

$$\boldsymbol{t'} = \frac{\boldsymbol{\sigma'_1 - \sigma'_3}}{\boldsymbol{2}}$$

$$\boldsymbol{e = void\ ratio}$$

$$\boldsymbol{v = specific\ volume = 1 + e}$$

$$\dot{\boldsymbol{\varepsilon}}_t = \frac{\boldsymbol{3}}{\boldsymbol{4}}\dot{\boldsymbol{\varepsilon}}_a$$

In space $\boldsymbol{p'x\,q'x\,v}$ points $\boldsymbol{C_1}$, $\boldsymbol{C_2}$ and $\boldsymbol{C_3}$ are on a critical state line associated to a given $\dot{\varepsilon}_t$.

The ***"viscosity jump"*** $\mathbb{V}$ was written as:

$$\mathbb{V} = \boldsymbol{\eta}(\boldsymbol{e})\,\frac{\boldsymbol{d(\varepsilon_1 - \varepsilon_3)}}{\boldsymbol{dt}} = \boldsymbol{\eta}(\boldsymbol{e})\,\dot{\boldsymbol{\gamma}}$$

If this was the case ⇨ the ***"viscosity jump"*** would be proportional to $\dot{\boldsymbol{\gamma}}$.

This is a feature which ***IS NOT OBSERVED*** experimentally.

Being so ⇨ the viscous resistance component should be written as a function $\boldsymbol{f}$ of the distortion rate $\dot{\boldsymbol{\gamma}}$ (or the shear strain rate $\dot{\boldsymbol{\varepsilon}}_t = \dot{\boldsymbol{\gamma}}/\mathbf{2}$), i.e.

$$\mathbb{V} = \boldsymbol{\eta}(\boldsymbol{e})\,\boldsymbol{f}(\dot{\boldsymbol{\gamma}})$$

The viscous resistance component should be written then as

$$\boldsymbol{\tau}_{\eta\alpha} = \boldsymbol{\eta}(\boldsymbol{e})\boldsymbol{f}\left[\frac{\boldsymbol{d(\varepsilon_1 - \varepsilon_3)}}{\boldsymbol{dt}}\right]\boldsymbol{sin\ 2\alpha} = \boldsymbol{\eta}(\boldsymbol{e})\,\boldsymbol{f}(\dot{\boldsymbol{\gamma}})\,\boldsymbol{sin\ 2\alpha} = \mathbb{V}\,\boldsymbol{sin\ 2\alpha}$$

$$\tau_{\eta\alpha} = \eta(e) f\left[\frac{d(\varepsilon_1 - \varepsilon_3)}{dt}\right] \sin 2\alpha = \eta(e)\, f(\dot{\gamma}) \sin 2\alpha = \mathbb{V} \sin 2\alpha$$

- In a $\overline{CIUCL}$ test carried out with $\dot{\gamma} = constant$, $f(\dot{\gamma}) = constant$. Thus the viscosity and friction ellipses, both still hold true. The minor axis of the viscosity ellipse should be written now as $2\,\mathbb{V} = 2\,\eta(e)\, f(\dot{\gamma})$.

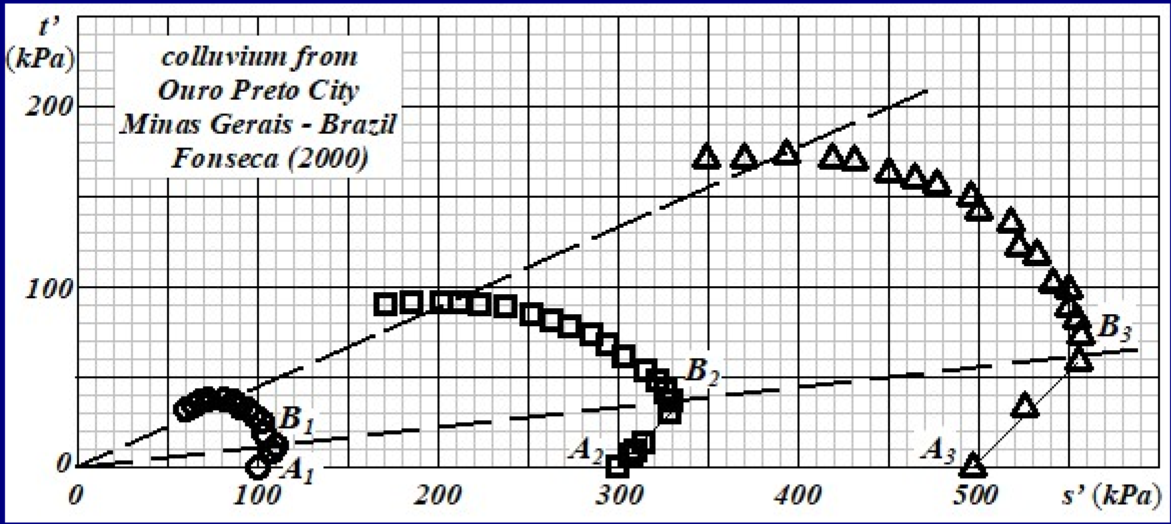
- Conclusion ⇨ the viscous resistance in a plastic soil is non-newtonian.

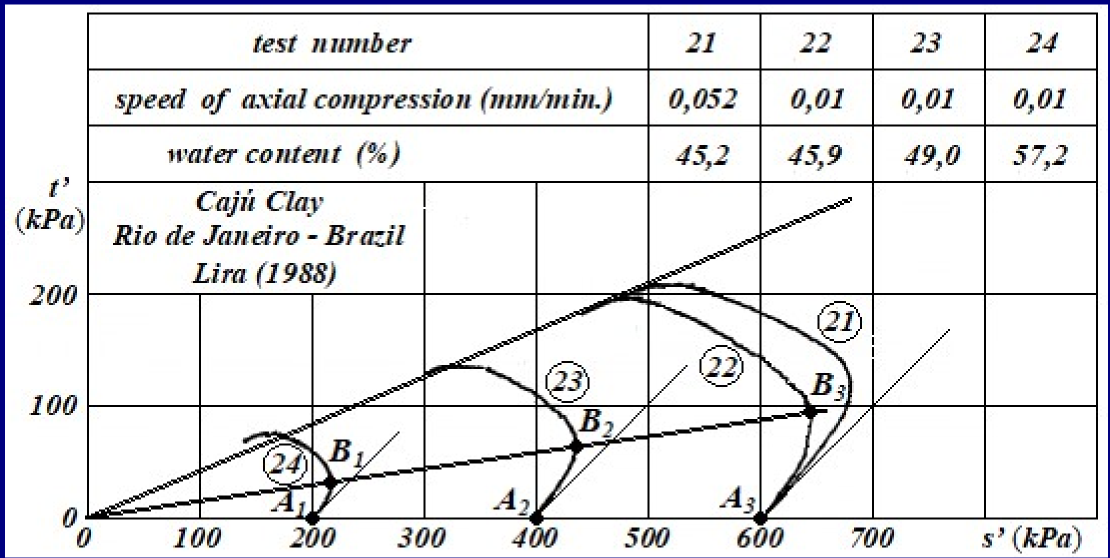
- On the other hand, the expression for $\tan \phi'_{emob}$,

$$\tan \phi'_{emob} = \frac{\left(\frac{\sigma'_1 - \sigma'_3}{2} - \mathbb{V}\right)}{\sqrt{\sigma'_1 \sigma'_3}} = \frac{(t' - \mathbb{V})}{\sqrt{s'^2 - t'^2}}$$

still holds true !

- For normally consolidated clays, the ***"viscosity jump"*** $\mathbb{V}$ is proportional to the isotropic consolidation stress $\boldsymbol{p'_e}$ (see examples below).



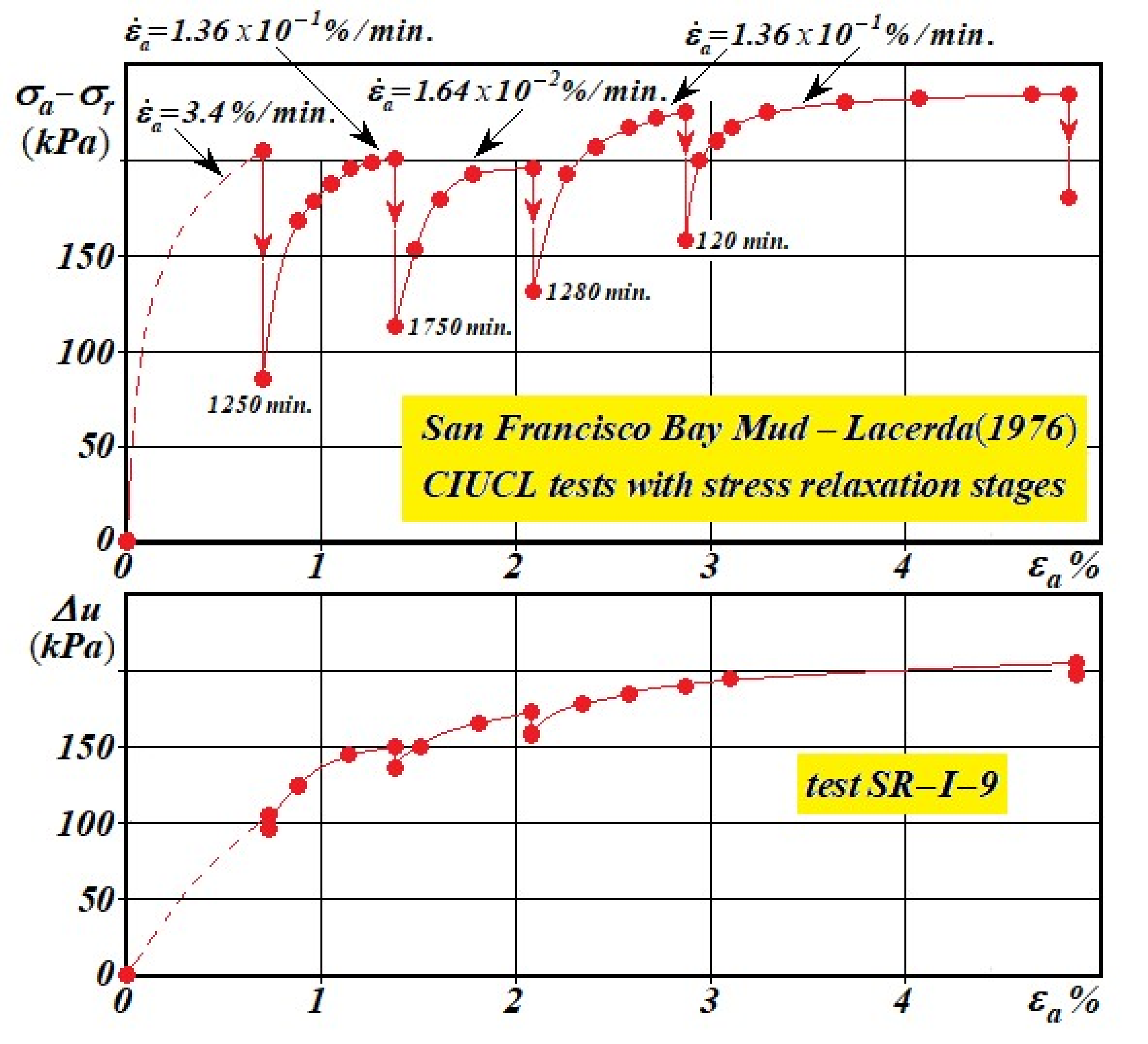
| test number | 21 | 22 | 23 | 24 |
|---|---|---|---|---|
| speed of axial compression (mm/min.) | 0,052 | 0,01 | 0,01 | 0,01 |
| water content (%) | 45,2 | 45,9 | 49,0 | 57,2 |



Based on experimental evidences

$$\mathbb{V} = \boldsymbol{\eta}(\boldsymbol{e})\, \boldsymbol{f}(\dot{\boldsymbol{\gamma}}) = \boldsymbol{C}_{\eta}(\dot{\boldsymbol{\gamma}})\, \boldsymbol{p}'_e$$

Strain rate effects – additional experimental evidences




$\overline{CIUCL}$ tests with varying axial strain rates ($\dot{\varepsilon}_a$) and stress relaxation stages.

Features observed by Lacerda (1976) → $\overline{CIUCL}$ tests on normally consolidated specimens of San Francisco Bay Mud with different axial strain rate values ($\dot{\varepsilon}_a$)

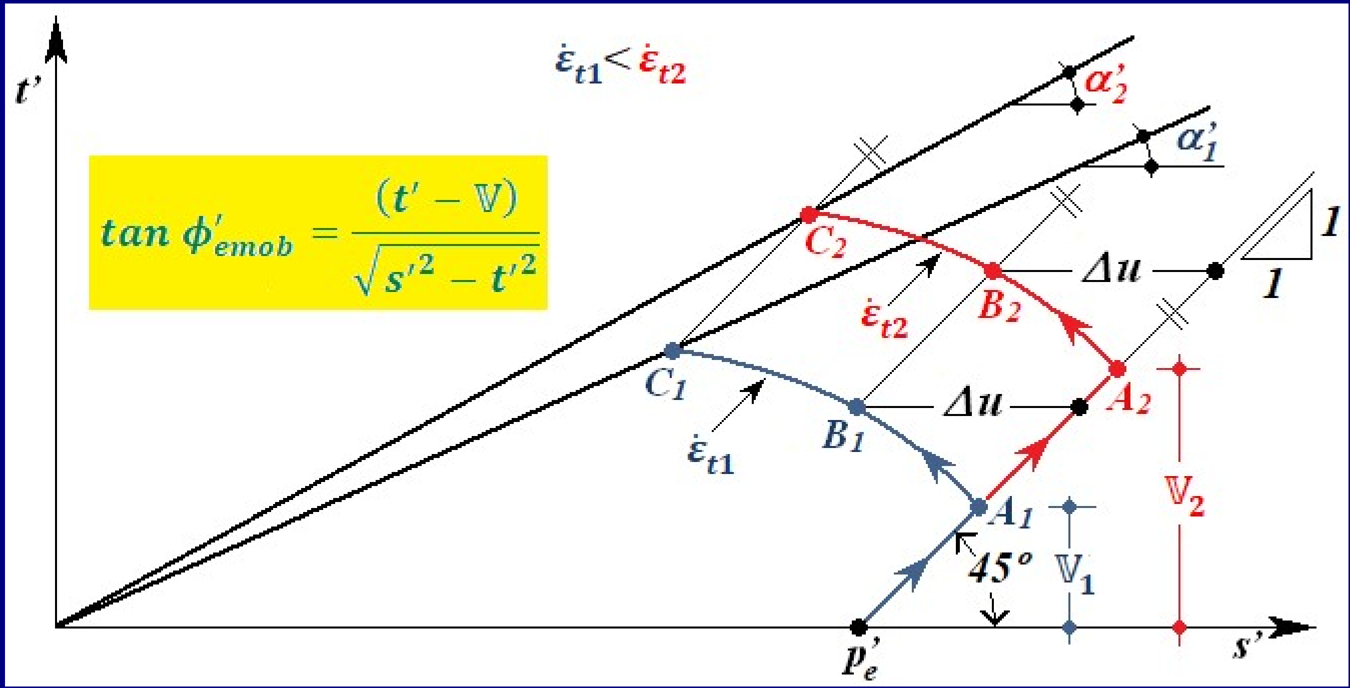
(a) Tests carried out with higher $\dot{\varepsilon}_a$ values present, for the same $\varepsilon_a$, higher values of the deviator stress $(\sigma_a - \sigma_r)$.

(b) For a fixed isotropic consolidation stress $p'_e$ the curve $\Delta u$ $x$ $\varepsilon_a$ (pore-pressure $x$ axial strain) is unique, irrespective of the axial strain rate $\dot{\varepsilon}_a$.

(c) Pore-pressure decrease during stress relaxation stages (when $\dot{\varepsilon}_a = 0$) are very small if compared to the deviator stress decrease.

These features allow to consider as a working hypothesis that pore-pressures do not depend on the axial strain rate ($\dot{\varepsilon}_a$) being dependent on the axial strain ($\varepsilon_a$) (or shear strain $\varepsilon_t$) and proportional to the isotropic consolidation stress $p'_e$.

$\overline{\boldsymbol{CIUCL}}$ tests carried out with different shear strain rates $\dot{\varepsilon}_{t1} < \dot{\varepsilon}_{t2}$ (remember $\dot{\varepsilon}_t = {}^3/_4\, \dot{\varepsilon}_a$)



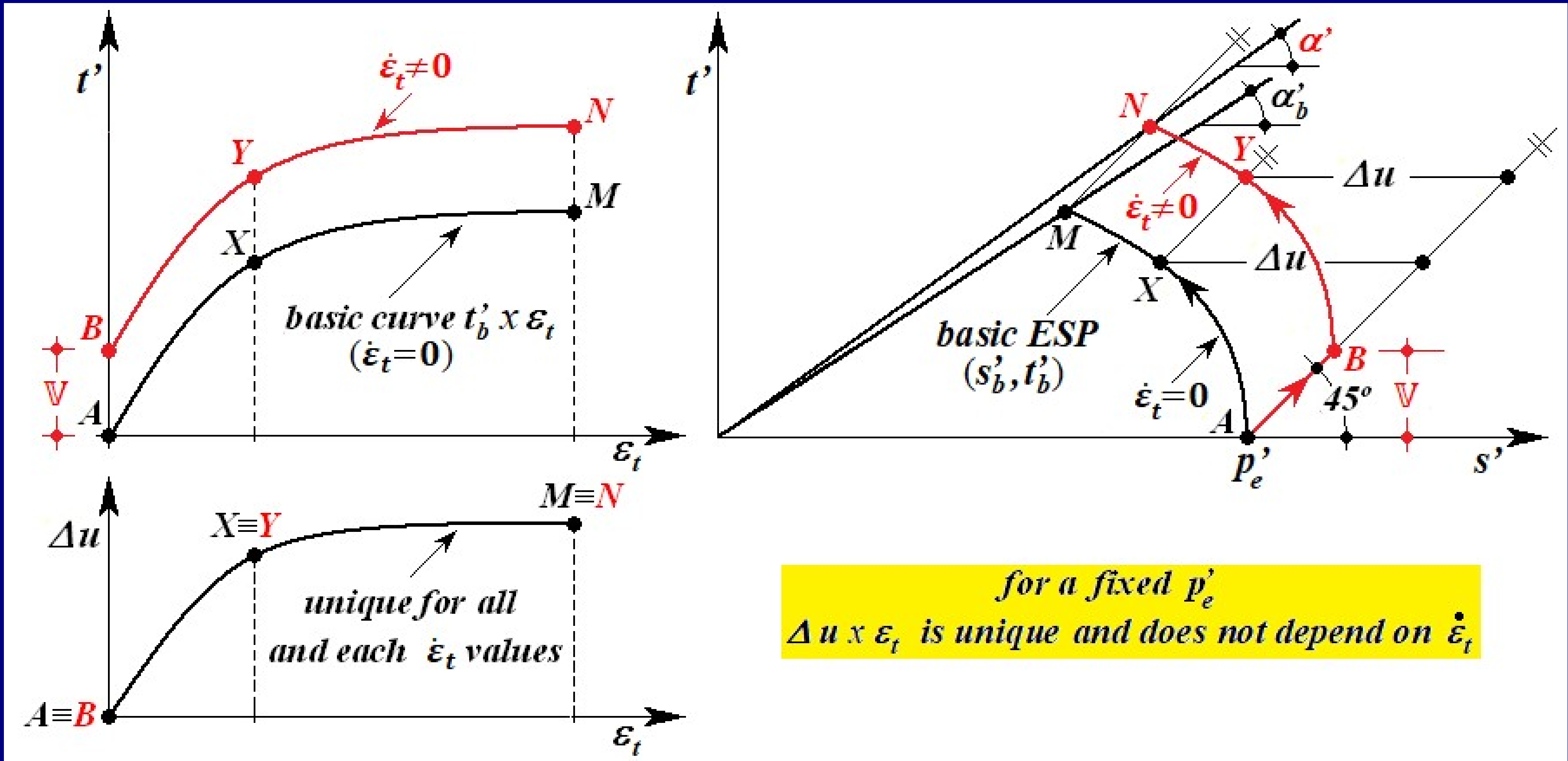

- At $A_1$ and $A_2$ ⇨ $\boldsymbol{\tan \phi'_{emob} = 0.}$

- At $C_1$ and $C_2$ , that represent failure ⇨ $\boldsymbol{\tan \phi'_{emob} = \tan \phi'_e}$.

- It will be assumed that all points on each straight line with slope 1:1, like $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$, which have the same $\boldsymbol{\Delta u}$ and the same $\boldsymbol{\varepsilon_t}$ but belong to distinct ***ESPs*** with different $\dot{\varepsilon}_t$ values, will also have the same $\boldsymbol{\phi'_{emob}}$. This statement will be called ***"PRINCIPLE 1"*** and its validity must be checked experimentally later.

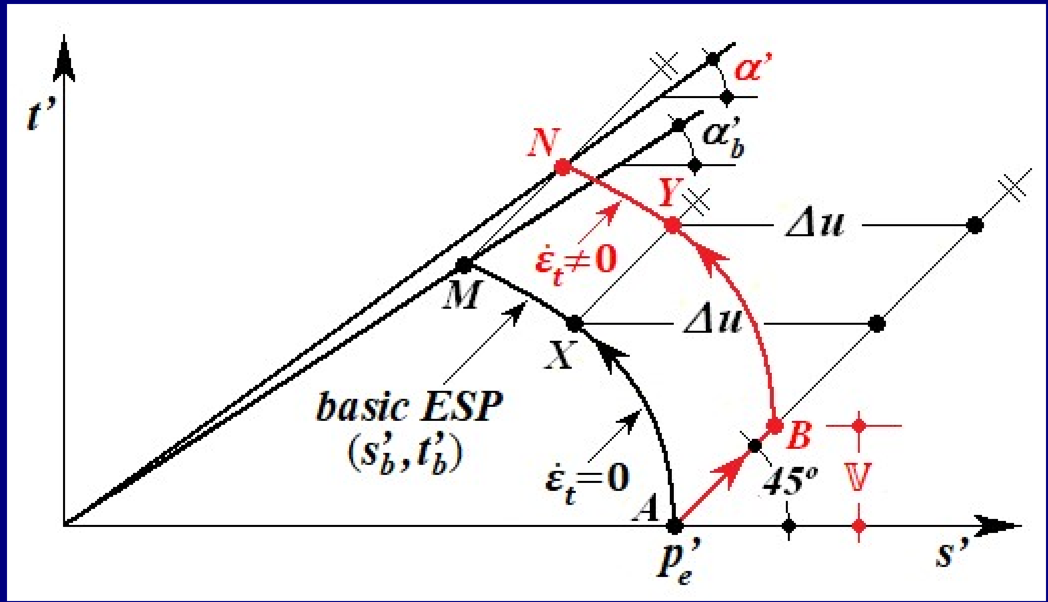
$t' \, x \, \varepsilon_t$, $\Delta u \, x \, \varepsilon_t$ and $t' x \, s'$ curves for a fixed $\dot{\varepsilon}_t \neq 0$

Basic curves $t'_b \, x \, \varepsilon_t$, $\Delta u \, x \, \varepsilon_t$ and $t'_b \, x \, s'_b$ ⇨ associated to $\dot{\varepsilon}_t = 0$.




- The test with $\dot{\varepsilon}_t \neq 0$ shows the viscosity jump ***AB*** corresponding to the instantaneous mobilization of the viscous resistance and afterwards follows the effective stress path ***BYN***, developing pore pressures $\boldsymbol{\Delta u}$ and shear strains $\boldsymbol{\varepsilon_t}$ until failure at point ***N.***
- In the test carried out with $\dot{\varepsilon}_t = 0$ there is no viscous resistance. There is only mobilization of friction resistance as the specimen is sheared. The basic curves ($\dot{\varepsilon}_t = 0$)are shown as ***AXM.***.

Relationship between coordinates $(s', t')$ of an ***ESP*** associated to an $\dot{\varepsilon}_t \neq 0$ and $(s'_b, t'_b)$ of a basic ***ESP*** associated to $\dot{\varepsilon}_t = 0$.




According to ***"PRINCIPLE 1"*** (which should be checked experimentally later)

$$\boldsymbol{tan\phi'_{emob}(Y) = \frac{t'(Y) - \mathbb{V}}{\sqrt{(s'(Y))^2 - (t'(Y))^2}} = tan\phi'_{emob}(X) = \frac{t'_b(X)}{\sqrt{\left(s'_b(X)\right)^2 - \left(t'_b(X)\right)^2}}}$$

and

$$\boldsymbol{\frac{t'(Y) - t'_b(X)}{s'(Y) - s'_b(X)} = 1}$$

Solving for $\boldsymbol{s'_b(X)}$ and $\boldsymbol{t'_b(X)}$

$$\boldsymbol{t'_b = \frac{(t' - \mathbb{V})^2}{(s' + t')}\left[1 + \frac{\sqrt{s'^2 + \mathbb{V}^2 - 2\mathbb{V}t'}}{(t' - \mathbb{V})}\right]}$$

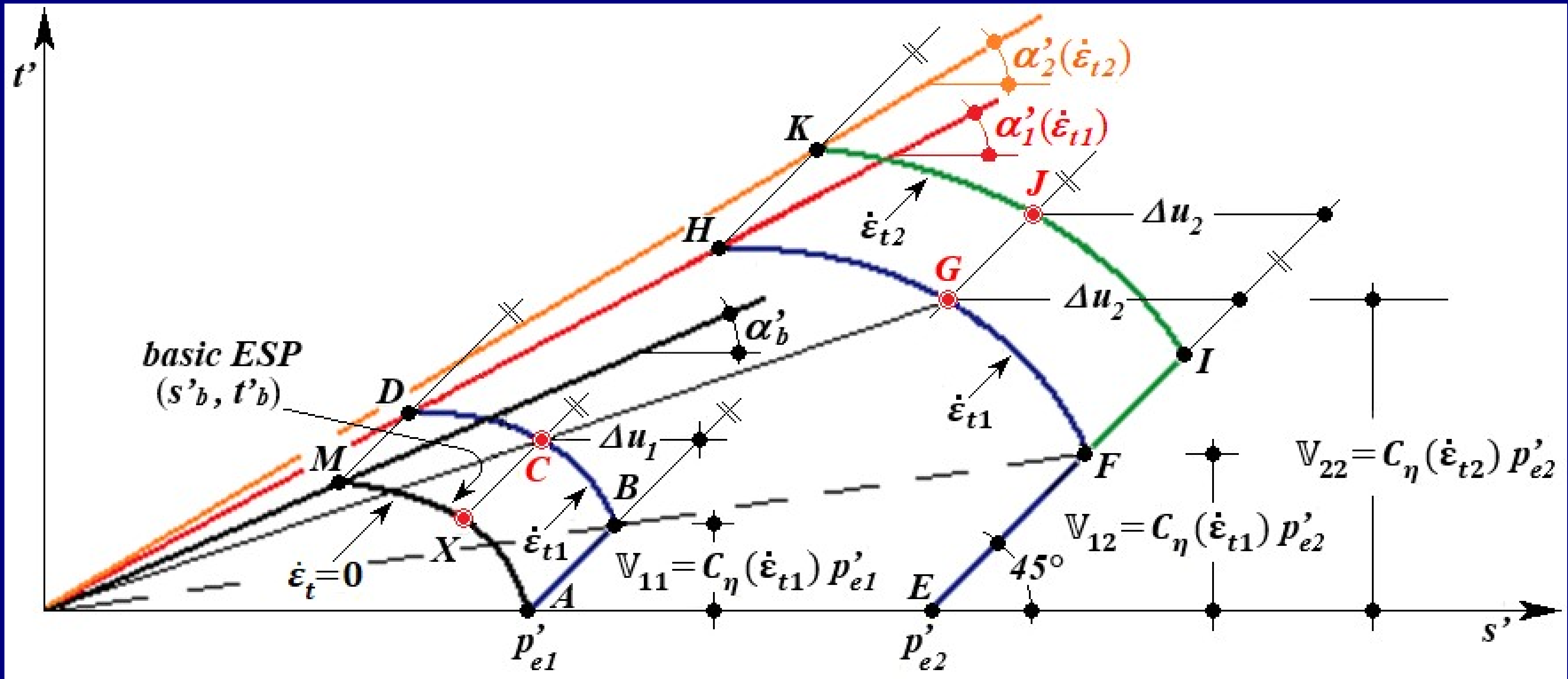
$$\boldsymbol{s'_b = (s' - t') + \frac{(t' - \mathbb{V})^2}{(s' + t')}\left[1 + \frac{\sqrt{s'^2 + \mathbb{V}^2 - 2\mathbb{V}t'}}{(t' - \mathbb{V})}\right]}$$

Conclusion ⇒ basic curve $\boldsymbol{t'_b}\ x\ \varepsilon_t$ and basic ***ESP*** coordinates $(\boldsymbol{s'_b, t'_b})$ can be obtained from $(\boldsymbol{s'}, \boldsymbol{t'})$ coordinates and viscous resistance $\mathbb{V}$ of an ***ESP*** associated to a test with $\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{t}} = \boldsymbol{0}$ . The $\boldsymbol{t'_b}\ x\ \varepsilon_t$ basic curve and basic ***ESP*** are free from viscous resistance, i.e. both are free from strain rate effects. Conversely, $(\boldsymbol{s'}, \boldsymbol{t'})$ for a given $\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{t}}$ can be obtained from $\mathbb{V}\ (\dot{\boldsymbol{\varepsilon}}_{\boldsymbol{t}})$ and $(\boldsymbol{s'_b, t'_b})$ by.

$$\boldsymbol{s'} = \mathbb{V} + \frac{\boldsymbol{s'_b}^2}{(\boldsymbol{s'_b + t'_b})} + \frac{\boldsymbol{t'_b}}{(\boldsymbol{s'_b + t'_b})}\sqrt{\boldsymbol{s'_b}^2 + 2\mathbb{V}(\boldsymbol{s'_b + t'_b})}$$

$$\boldsymbol{t'} = \mathbb{V} + \frac{\boldsymbol{t'_b}^2}{(\boldsymbol{s'_b + t'_b})} + \frac{\boldsymbol{t'_b}}{(\boldsymbol{s'_b + t'_b})}\sqrt{\boldsymbol{s'_b}^2 + 2\mathbb{V}(\boldsymbol{s'_b + t'_b})}$$

Global normalization and basic curves

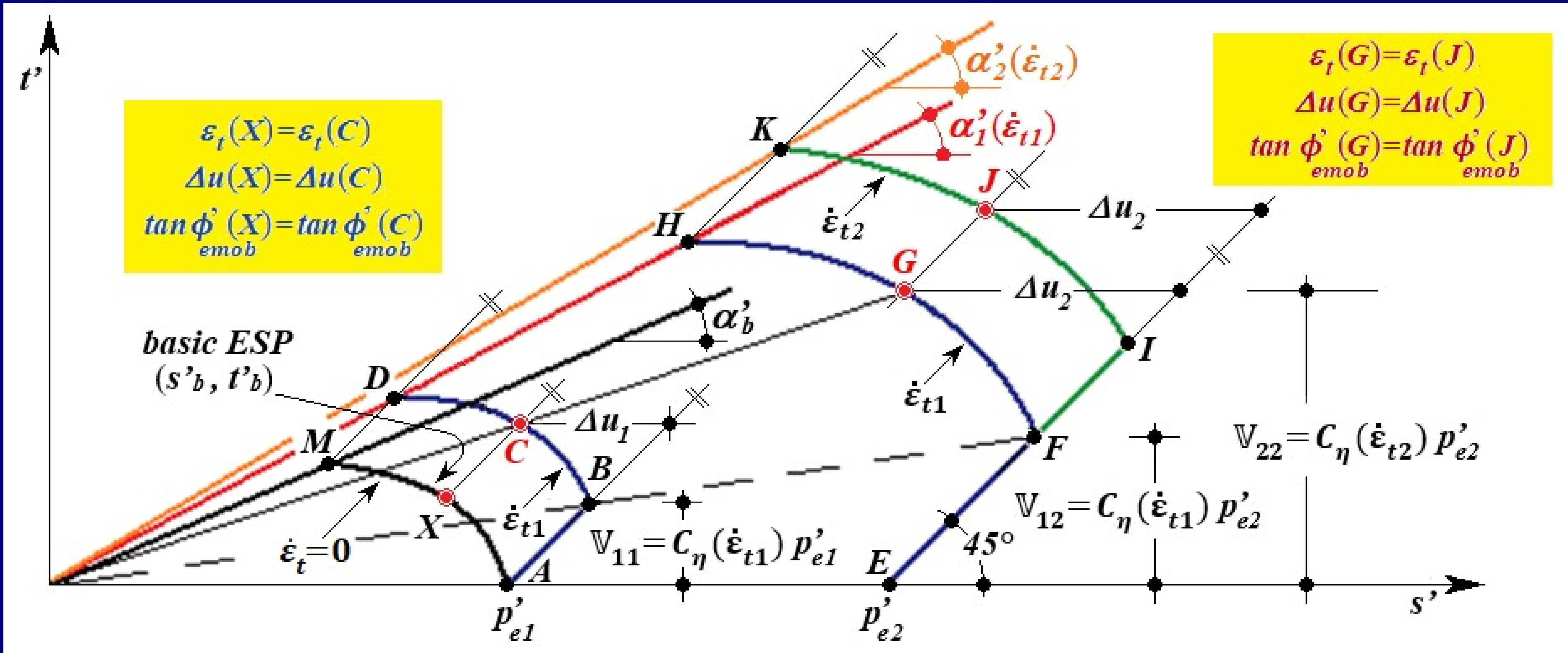



Both ***ABCD*** and ***AXM*** effective stress paths depart from the same $p'_{e1}$ with different $\dot{\varepsilon}_t$ values. As $\Delta u$ depend on $\varepsilon_t$ and $p'_e$ but do not depend on $\dot{\varepsilon}_t$ ➔ $\Delta u(C) = \Delta u(X) = \Delta u_1$ *and* $\varepsilon_t(C) = \varepsilon_t(X)$.

Thus, according to ***"PRINCIPLE 1"*** ➔ $\tan\phi'_{emob}(C) = \tan\phi'_{emob}(X)$.

Both ***EFIJK*** and ***EFGH*** effective stress paths depart from the same $p'_{e2}$ with different $\dot{\varepsilon}_t$ values. As $\Delta u$ depend on $\varepsilon_t$ and $p'_e$ but do not depend on $\dot{\varepsilon}_t$ ➔ $\Delta u(J) = \Delta u(G) = \Delta u_2$ *e* $\varepsilon_t(J) = \varepsilon_t(G)$

Thus according to ***"PRINCIPLE 1"*** ➔ $\tan\phi'_{emob}(J) = \tan\phi'_{emob}(G)$.

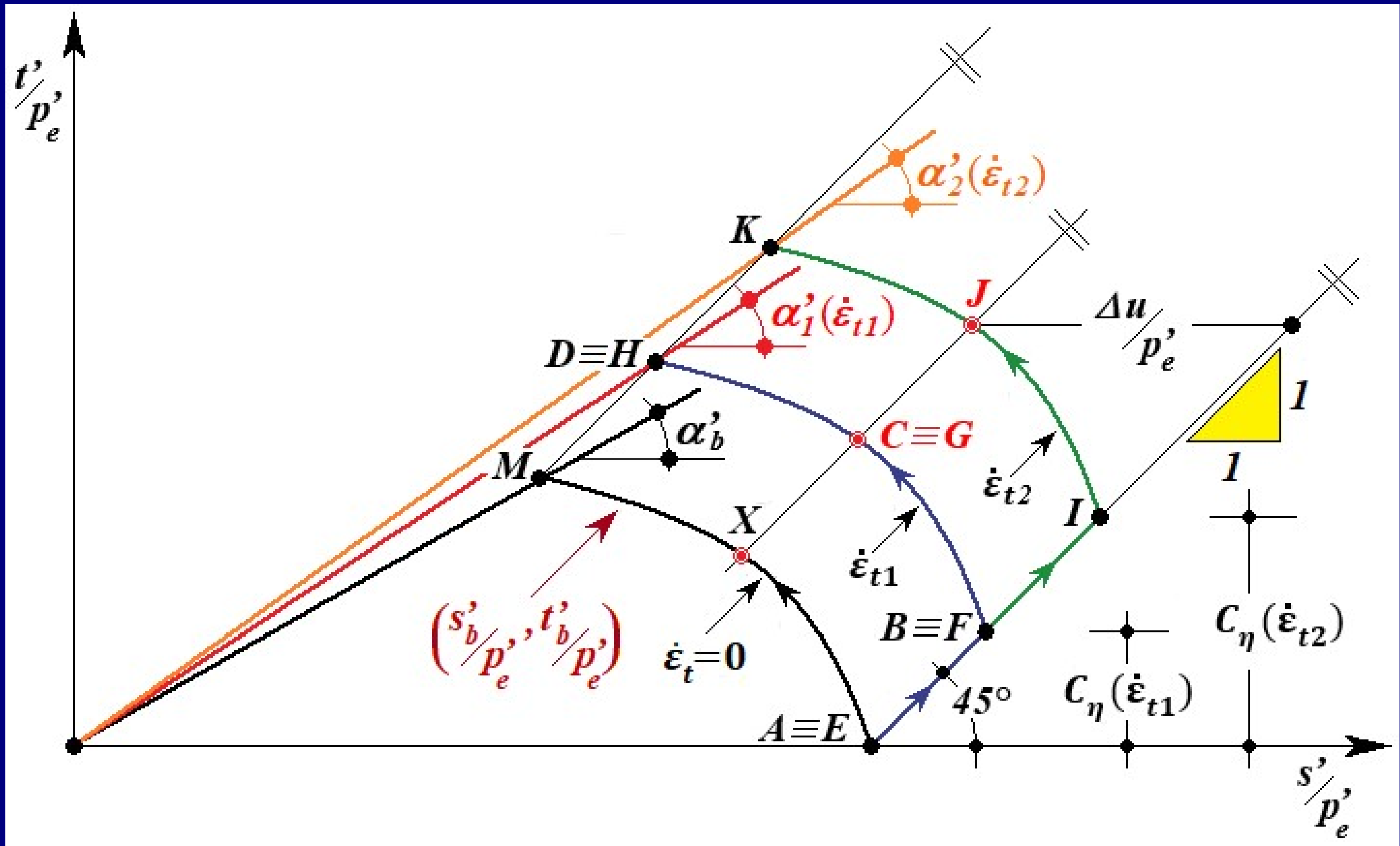



$$\frac{s'(C)}{p'_{e1}}=\frac{s'(G)}{p'_{e2}} \quad \text{e} \quad \frac{t'(C)}{p'_{e1}}=\frac{t'(G)}{p'_{e2}}$$

$$\tan\phi'_{emob}(G)=\frac{t'(G)-\mathbb{V}_{12}}{\sqrt{(s'(G))^2-(t'(G))^2}}=\frac{\frac{t'(G)}{p'_{e2}}-\frac{\mathbb{V}_{12}}{p'_{e2}}}{\sqrt{\frac{(s'(G))^2}{(p'_{e2})^2}-\frac{(t'(G))^2}{(p'_{e2})^2}}}=\frac{\frac{t'(C)}{p'_{e1}}-\frac{\mathbb{V}_{11}}{p'_{e1}}}{\sqrt{\frac{(s'(C))^2}{(p'_{e1})^2}-\frac{(t'(C))^2}{(p'_{e1})^2}}}=\tan\phi'_{emob}(C)$$

**Conclusion** ➡ $\tan\phi'_{emob}(G)=\tan\phi'_{emob}(J)=\tan\phi'_{emob}(C)=\tan\phi'_{emob}(X)$

Points resting on the same straight line with slope 1:1 crossing normalized effective stress paths $\left({s'}/{p'_e}\ x\ {t'}/{p'_e}\right)$ with different strain rates $\dot{\varepsilon}_t$ will have the same values of $\boldsymbol{\varepsilon_t}$, $\boldsymbol{\Delta u / p'_e}$ and $\boldsymbol{tan\ \phi'_{emob}}$.




This conclusion leads to the following consequences:

In $\overline{\boldsymbol{CIUCL}}$ tests carried out on a normally consolidated plastic soil the curves $\boldsymbol{\Delta u/p'_e}\ x\ \boldsymbol{\varepsilon_t}, \boldsymbol{tan\,\phi'_{emob}}\ x\ \boldsymbol{\varepsilon_t}$, $\boldsymbol{t'_b/p'_e}\ x\ \boldsymbol{\varepsilon_t}$ and the normalized basic effective stress path $(\boldsymbol{s'_b/p'_e}, \boldsymbol{t'_b/p'_e})$ (corresponding to $\dot{\varepsilon}_t = 0$) are unique and properties of the soil.

**Validity of proposed model applied to *“San Francisco Bay Mud”* (*Lacerda, 1976*)**

Samples obtained with a 5” diameter and 300 to 450 mm length piston samplers, taken in the Hamilton Air Base region (California) between the depths of 5.20 m and 7.60 m.

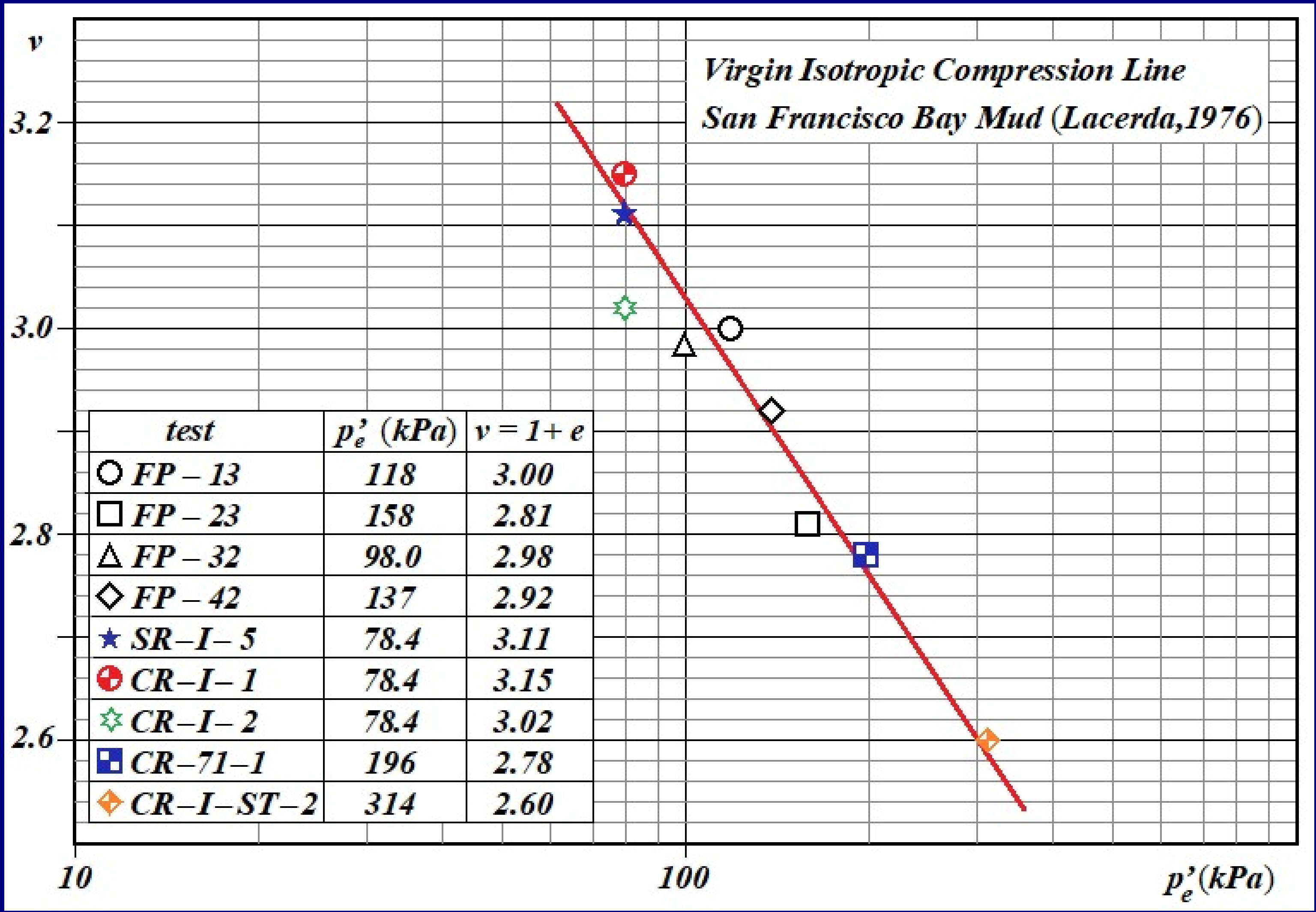
**Table 1**. Characterization test results of San Francisco Bay Mud Samples (Lacerda,1976)

| natural water content $w$ % | liquid limit $w_l$ % | plastic limit $w_p$ % | plasticity index PI % | specific gravity G | clay fraction % < 2μm | activity |
|---|---|---|---|---|---|---|
| 88 to 93 | 88 to 90 | 35 to 44 | 45 to 55 | 2.75 | 60 | 0.83 |

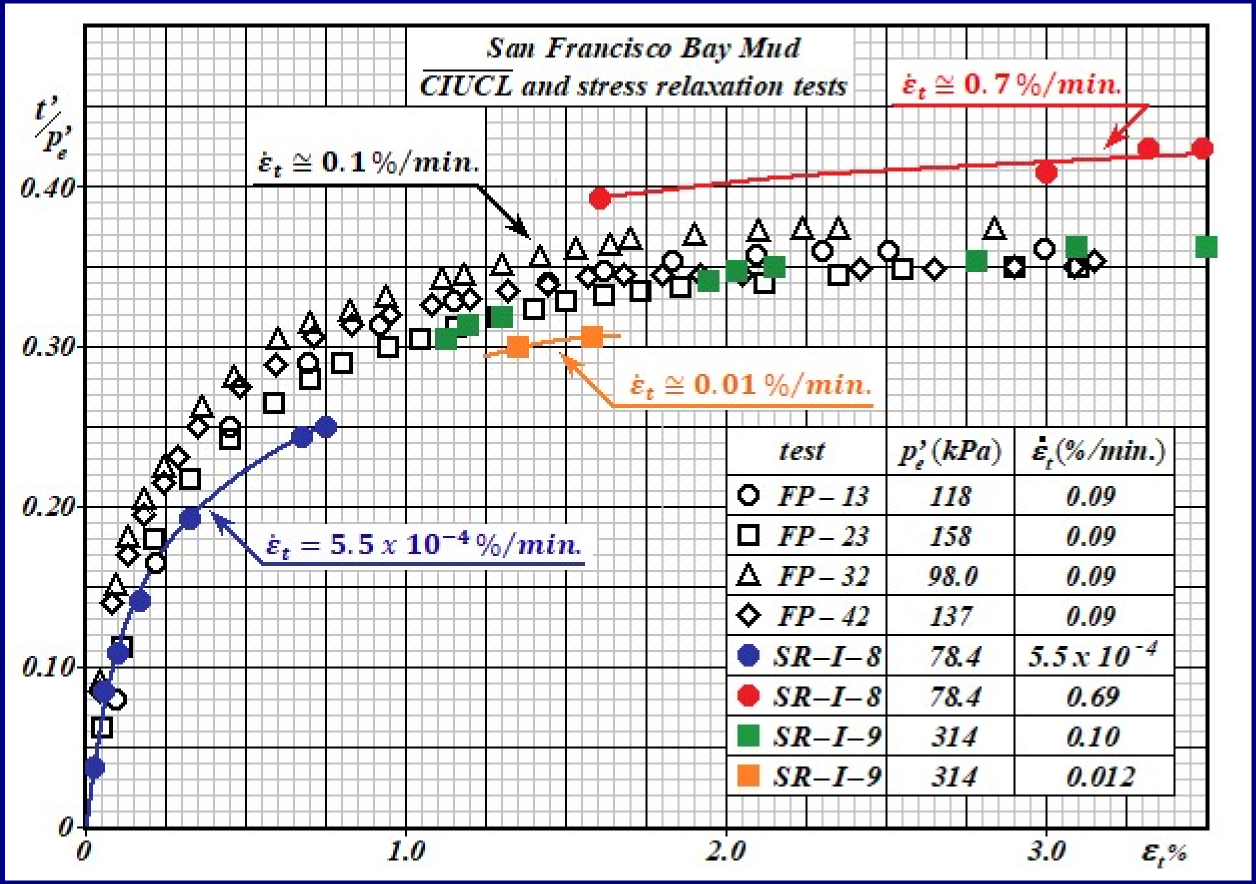
**Table 2**. Selected tests from Lacerda (1976) analyzed in this article

| Test | description | spec. vol. after isotropic compression $v$ | isotropic compression stress $p'_e$ (kPa) | strain rate $\dot{\varepsilon}_t$ $(\%/min.)$ |
|---|---|---|---|---|
| FP-13 | $\overline{CIUCL}$ | 3.00 | 118 | 0.09 |
| FP-23 | $\overline{CIUCL}$ | 2.81 | 157 | 0.09 |
| FP-32 | $\overline{CIUCL}$ | 2.98 | 98.1 | 0.09 |
| FP-42 | $\overline{CIUCL}$ | 2.92 | 137 | 0.09 |
| SR-I-5 | stress relax. | 3.11 | 78.4 | 1.15 |
| SR-I-8 | stress relax. | not inform. | 78.4 | $5.5 \times 10^{-4}$ |
| SR-I-9 | stress relax. | not inform. | 314 | 0.10 |
| CR-I-1 | und. creep | 3.15 | 78.4 | variable |
| CR-I-2 | und. creep | 3.02 | 78.4 | variable |
| CR-71-1 | und. creep | 2.78 | 196 | variable |
| CR-I-ST-2 | und. step creep | 2.60 | 314 | variable |

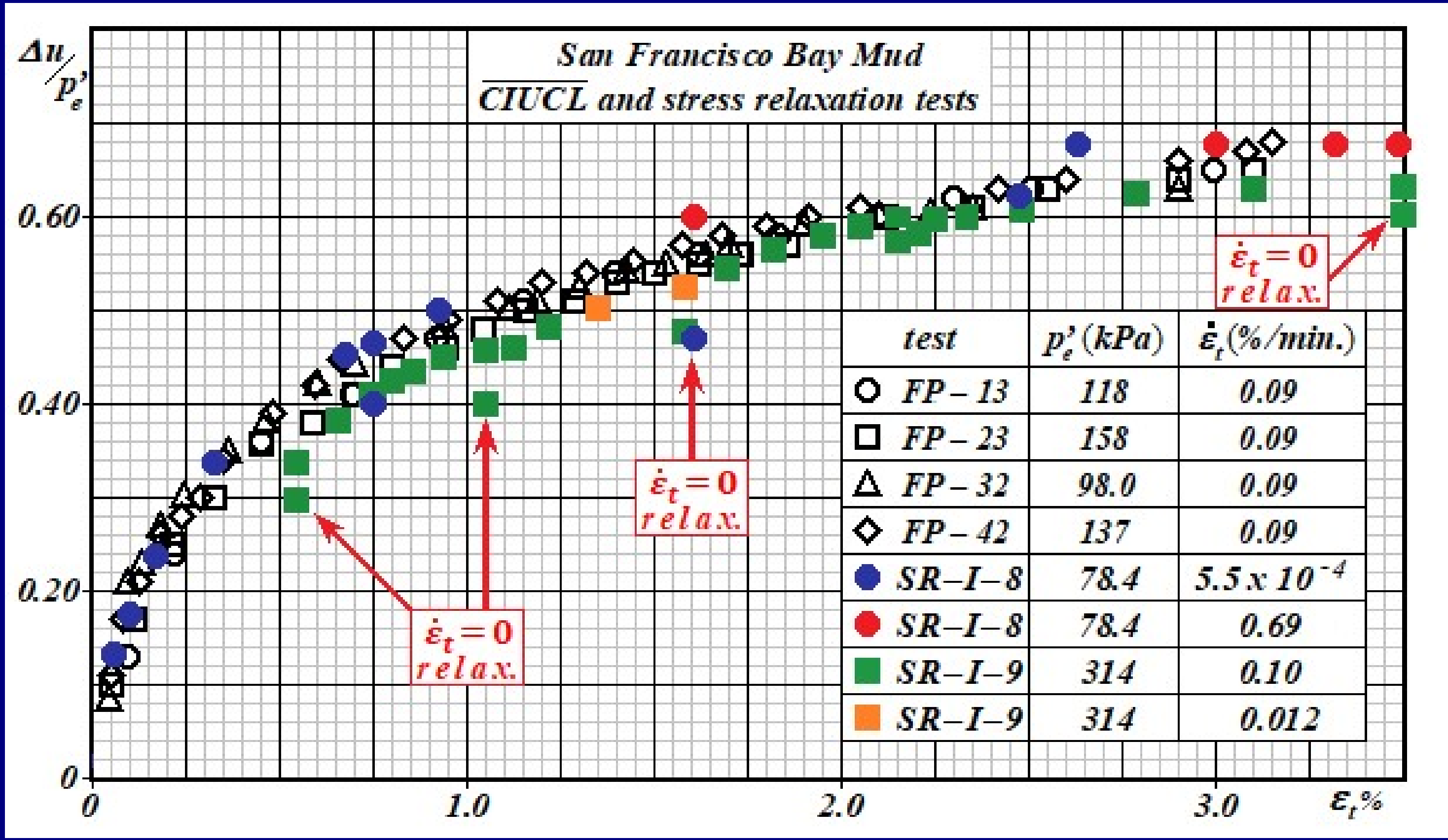
Virgin isotropic compression line – ***VICL*** – San Francisco Bay Mud (Lacerda,1976)



Superposition of $(t'/p'_e)\ x\ \varepsilon_t$ graphs from $\overline{CIUCL}$ tests with $\dot{\varepsilon}_t \cong 0.1\ \%/min.$ and $\overline{CIUCL}$ tests with varying strain rates and stress relaxation stages - San Francisco Bay Mud (from Lacerda, 1976)

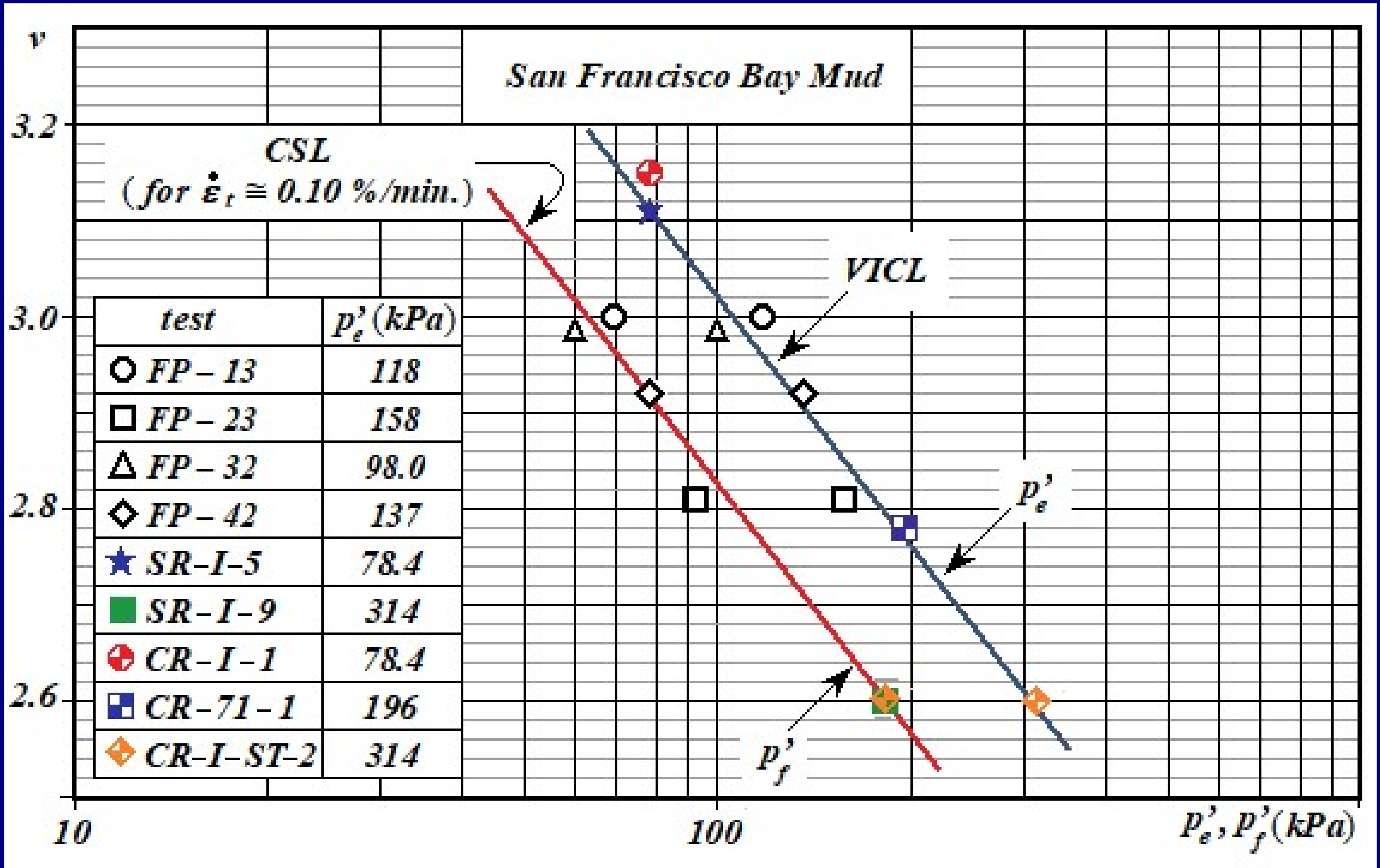


Superposition of $(\Delta u/p'_e)$ $x$ $\varepsilon_t$ graphs from $\overline{CIUCL}$ with $\dot{\varepsilon}_t \cong 0.1$ $\%/min.$ and $\overline{CIUCL}$ with varying strain rates and stress relaxation stages - San Francisco Bay Mud (from Lacerda, 1976)



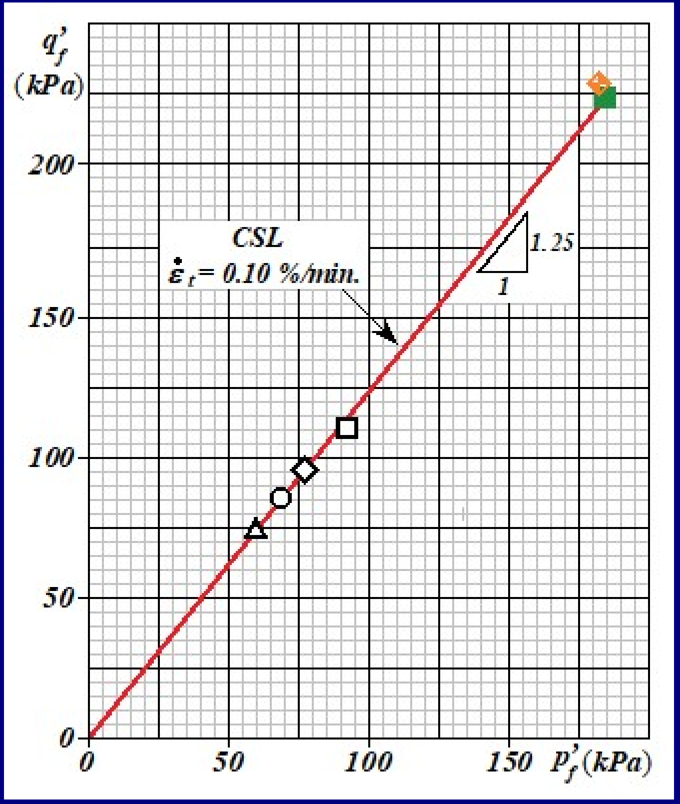

Pore-pressures can be considered to be not dependent on the strain rate $\dot{\varepsilon}_t$.

Virgin isotropic compression line (*VICL*) and critical state line ***(CSL)*** corresponding to $\dot{\varepsilon}_t \cong \mathbf{0.10}\ \%/\boldsymbol{min}.$ - San Francisco Bay Mud (data from Lacerda, 1976)



Critical state line on a $q' \, x \, p'$ plot for the strain rate $\dot{\varepsilon}_t \cong 0.10\ \%/min$.

"San Francisco Bay Mud" (data from Lacerda, 1976).



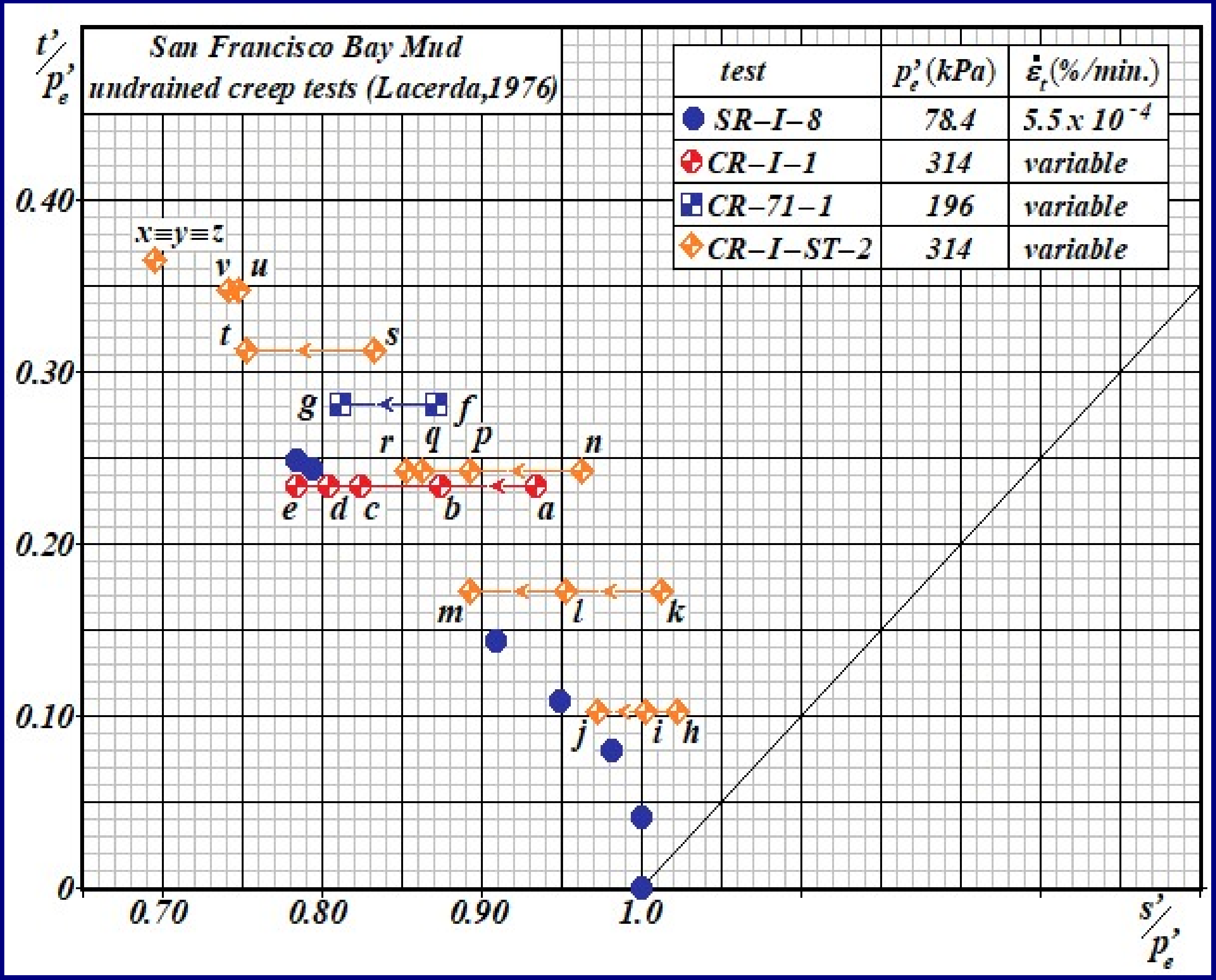

With $M = 1.25$, $\phi'$ value for the normally consolidated condition is $\cong 31°$.

However, $\phi'_e$ value can only be known after determining the value of $\mathbb{V} = C_\eta(\dot{\varepsilon}_t)\, p'_e$ .

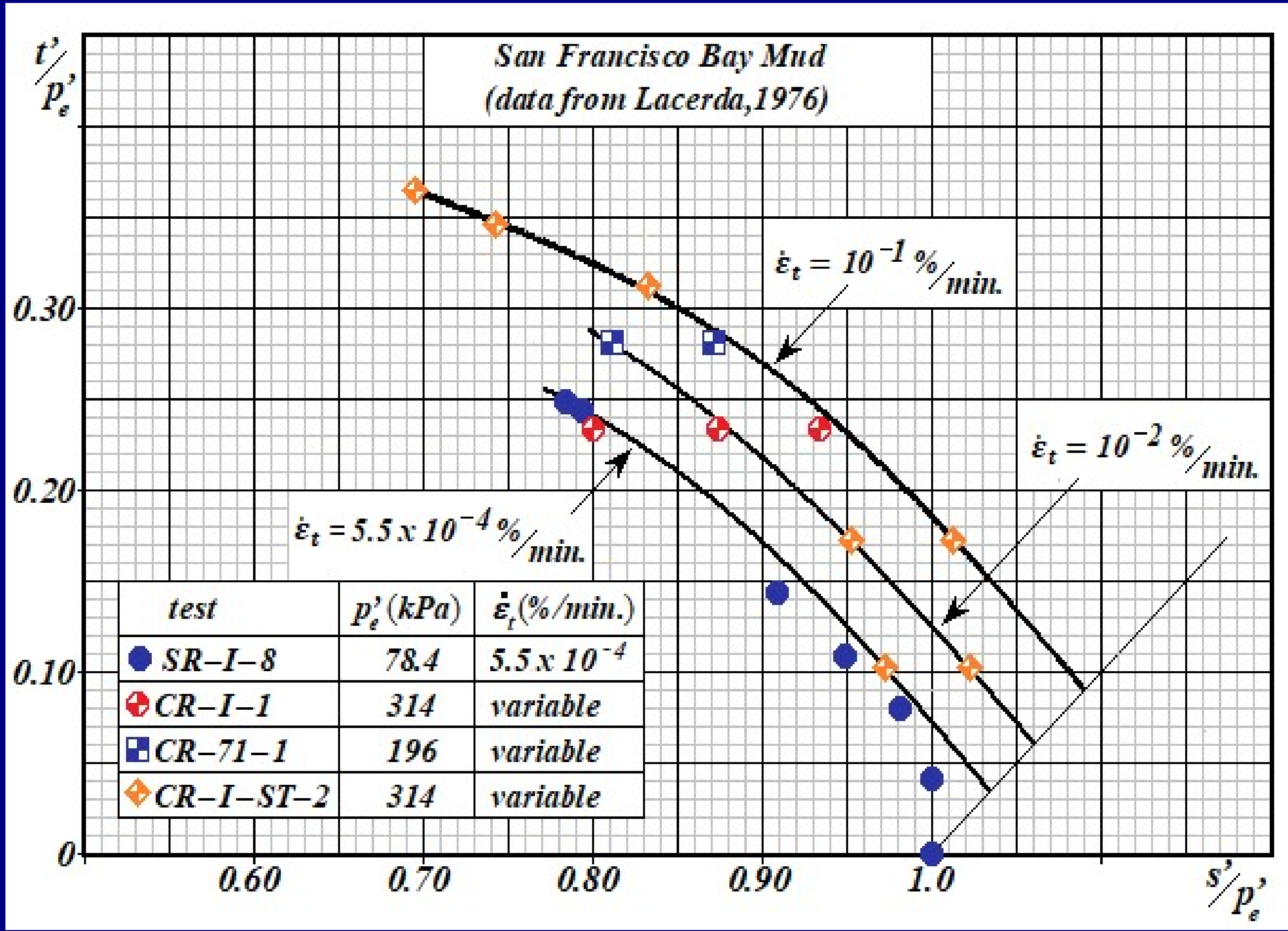
Determination of $\mathbb{V}$ ⟹ use of undrained creep tests results.

$C_\eta(\dot{\varepsilon}_t)$ and $\phi'_e$ are determined via undrained creep tests.

Undrained creep tests ⇨ *t'* is kept constant with time and $\varepsilon_t, \Delta u$ and $\dot{\varepsilon}_t$ are measured.



Appraisal of $\boldsymbol{C_\eta(\dot{\varepsilon}_t)}$ via undrained creep tests – ***ESPs*** fitted to points of equal strain rates $\boldsymbol{\dot{\varepsilon}_t}$




$$C_\eta(5.5\ x\ 10^{-4}\ \%/min.) \cong 0.035$$

$$C_\eta(\ 10^{-2}\ \%/min.) \cong 0.06$$

$$C_\eta(\ 10^{-1}\ \%/min.) \cong 0.09$$

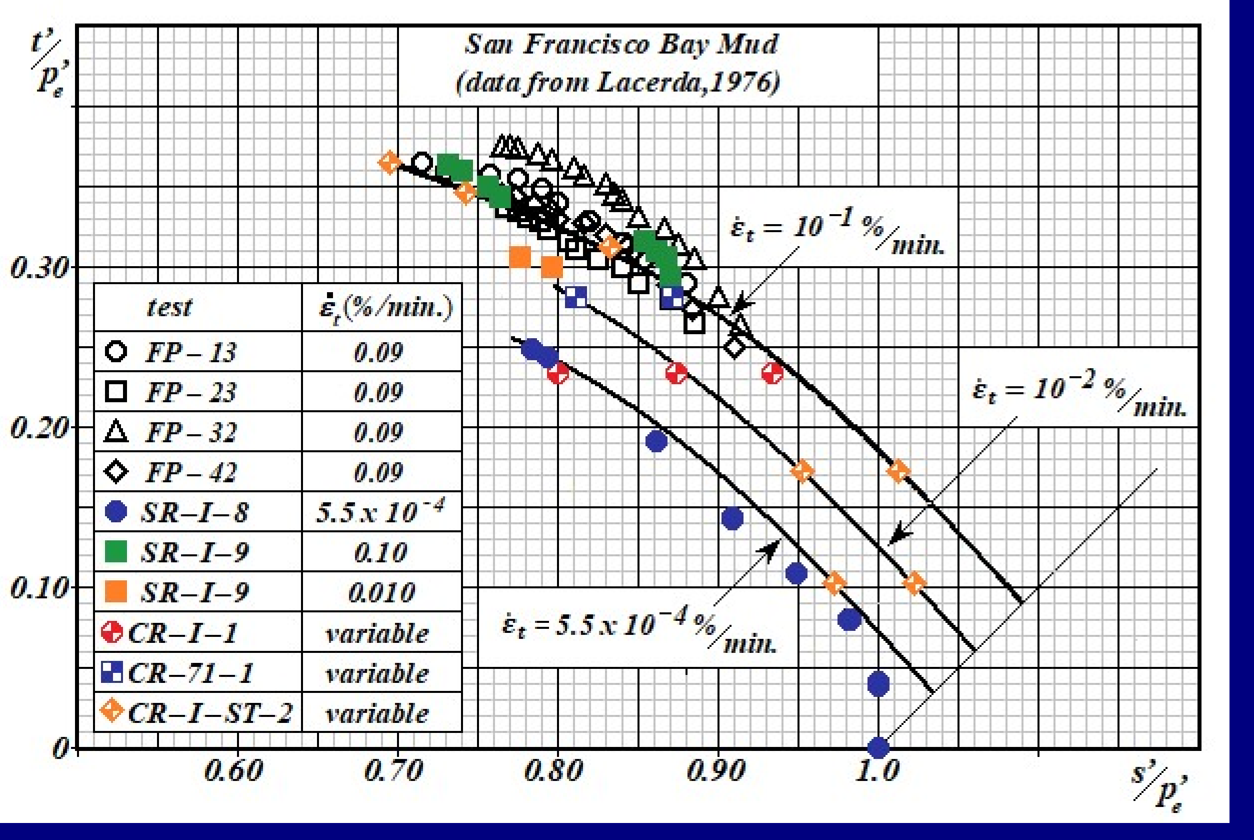
Assuming $C_{\eta}(0.1\ \%/min.) \cong 0.09$ and failure occurring at $(s'_f/p'_e\,, t'_f/p'_e) = (0.70, 0.36)$ for tests carried out with $\dot{\varepsilon}_t$ equal to ***0.1 %/min*** and ***0.09 %/min.*** , $tan\ \phi'_e$ can be evaluated as

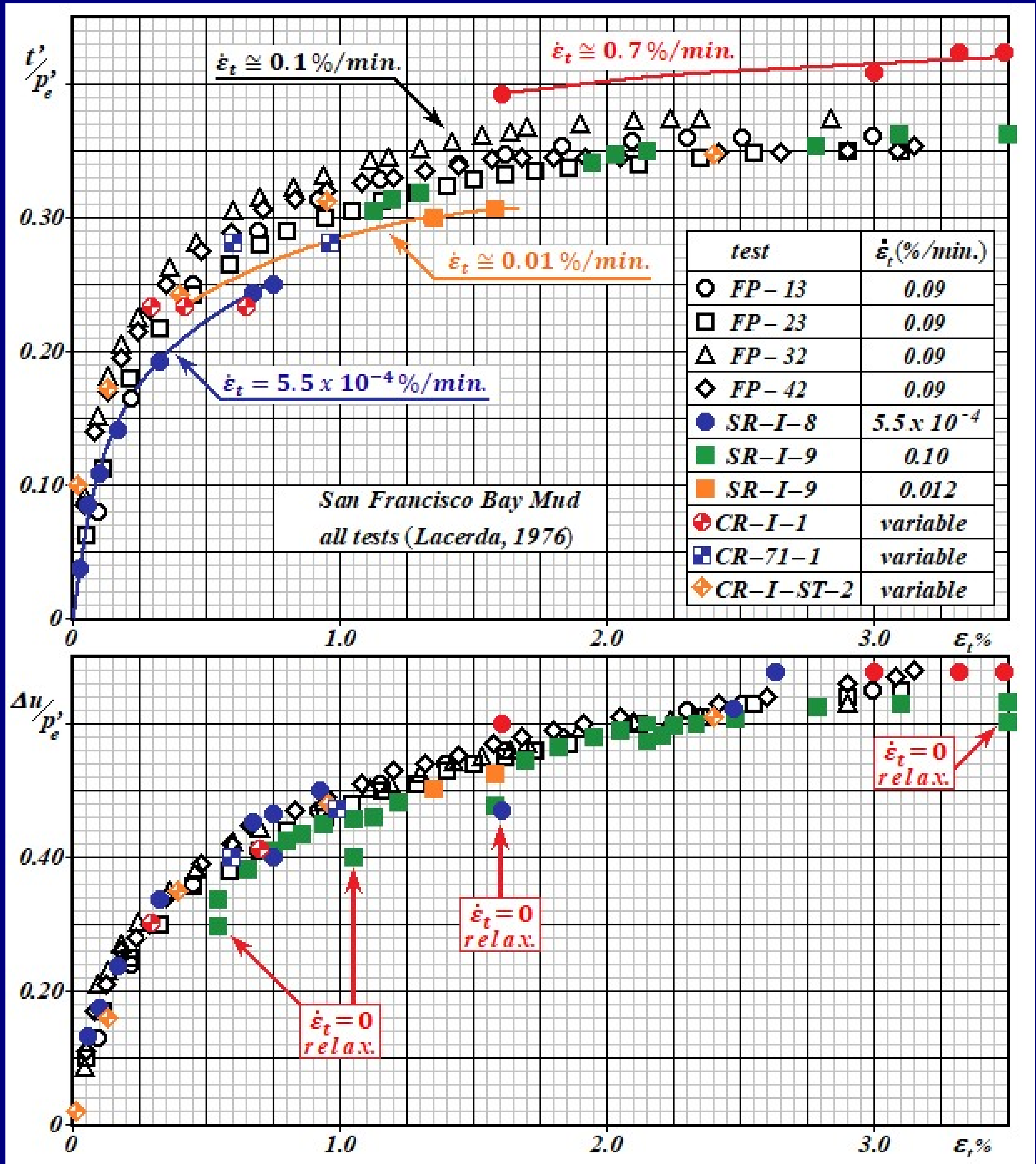
$$tan\ \phi'_e = \frac{\left(t'_f/p'_e - C_{\eta}(\dot{\varepsilon}_t)\right)}{\sqrt{\left(s'_f/p'_e\right)^2 - \left(t'_f/p'_e\right)^2}} = \frac{0.36 - 0.09}{\sqrt{0.70^2 - 0.36^2}} = 0.45 \Rightarrow \phi'_e \cong 24°$$

As far as normally consolidated San Francisco Bay Mud is concerned, the component of shear strength due to friction can be calculated using the Hvorslev true angle of friction $\phi'_e \cong 24°$.

In the shear strength values of normally consolidated San Francisco Bay Mud calculated with $\phi' \cong 31°$ there is embedded a viscous resistance corresponding to $\dot{\varepsilon}_t \cong$ ***0.1 %/min.***

Effective stress paths for all tests with different strain rates.






*All tests*

*(including*

*creep tests)*

Checking the conclusion

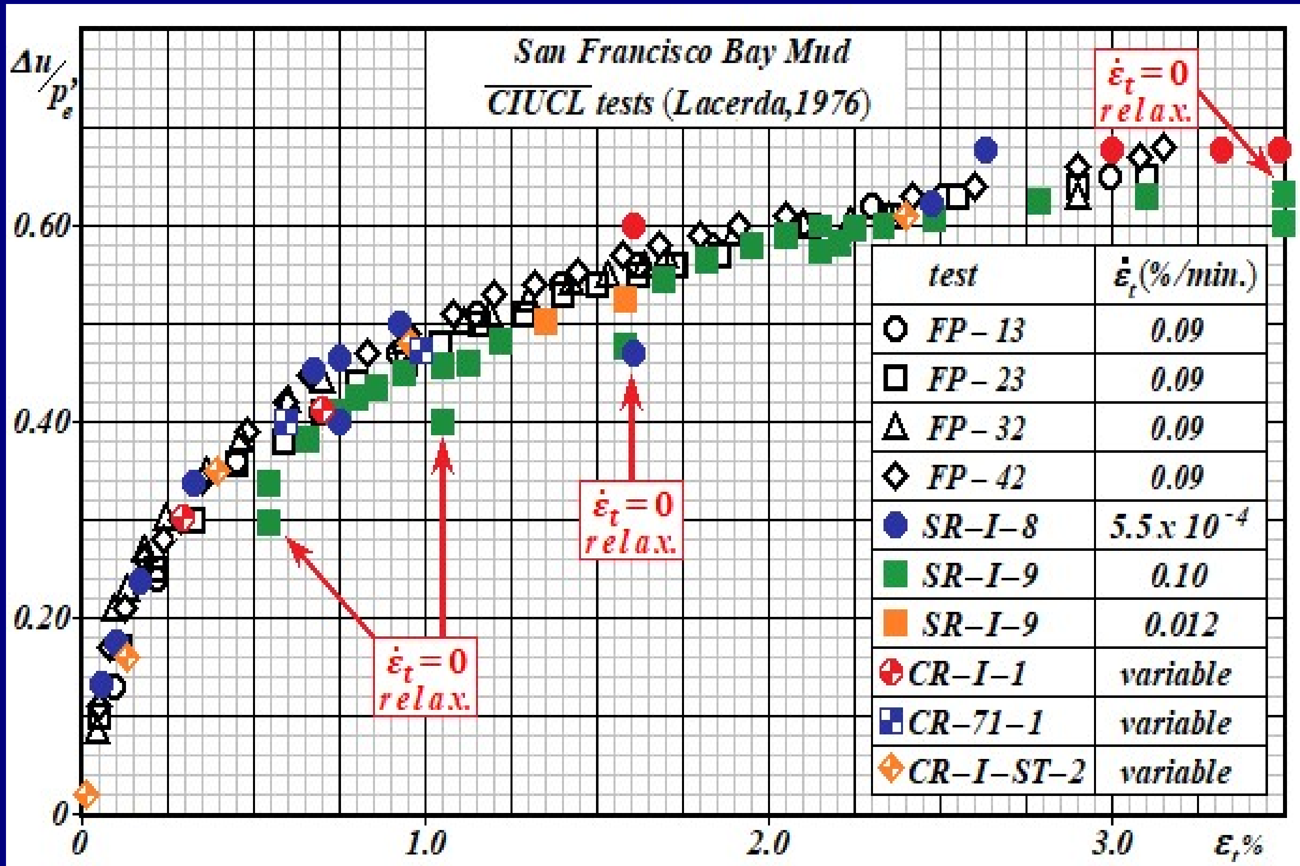
In $\overline{\boldsymbol{CIUCL}}$ tests carried out on normally consolidated plastic soils the curves $\boldsymbol{\Delta u/p'_e}\ x\ \boldsymbol{\varepsilon_t}$, $\boldsymbol{t'_b/p'_e}\ x\ \boldsymbol{\varepsilon_t}$, $\boldsymbol{tan\ \phi'_{emob}}\ x\ \boldsymbol{\varepsilon_t}$, and the normalized basic effective stress path $(\boldsymbol{s'_b/p'_e}\,,\boldsymbol{t'_b/p'_e})$ (for $\dot{\varepsilon}_t = 0$) are unique and properties of a soil.

$$\frac{t'_b}{p'_e} = \frac{\left(\dfrac{t'}{p'_e} - \dfrac{\mathbb{V}}{p'_e}\right)^2}{\left(\dfrac{s'}{p'_e} + \dfrac{t'}{p'_e}\right)}\left[1 + \frac{\sqrt{\dfrac{s'^2}{p'^2_e} + \dfrac{\mathbb{V}^2}{p'^2_e} - \dfrac{2\mathbb{V}t'}{p'^2_e}}}{\left(\dfrac{t'}{p'_e} - \dfrac{\mathbb{V}}{p'_e}\right)}\right]$$

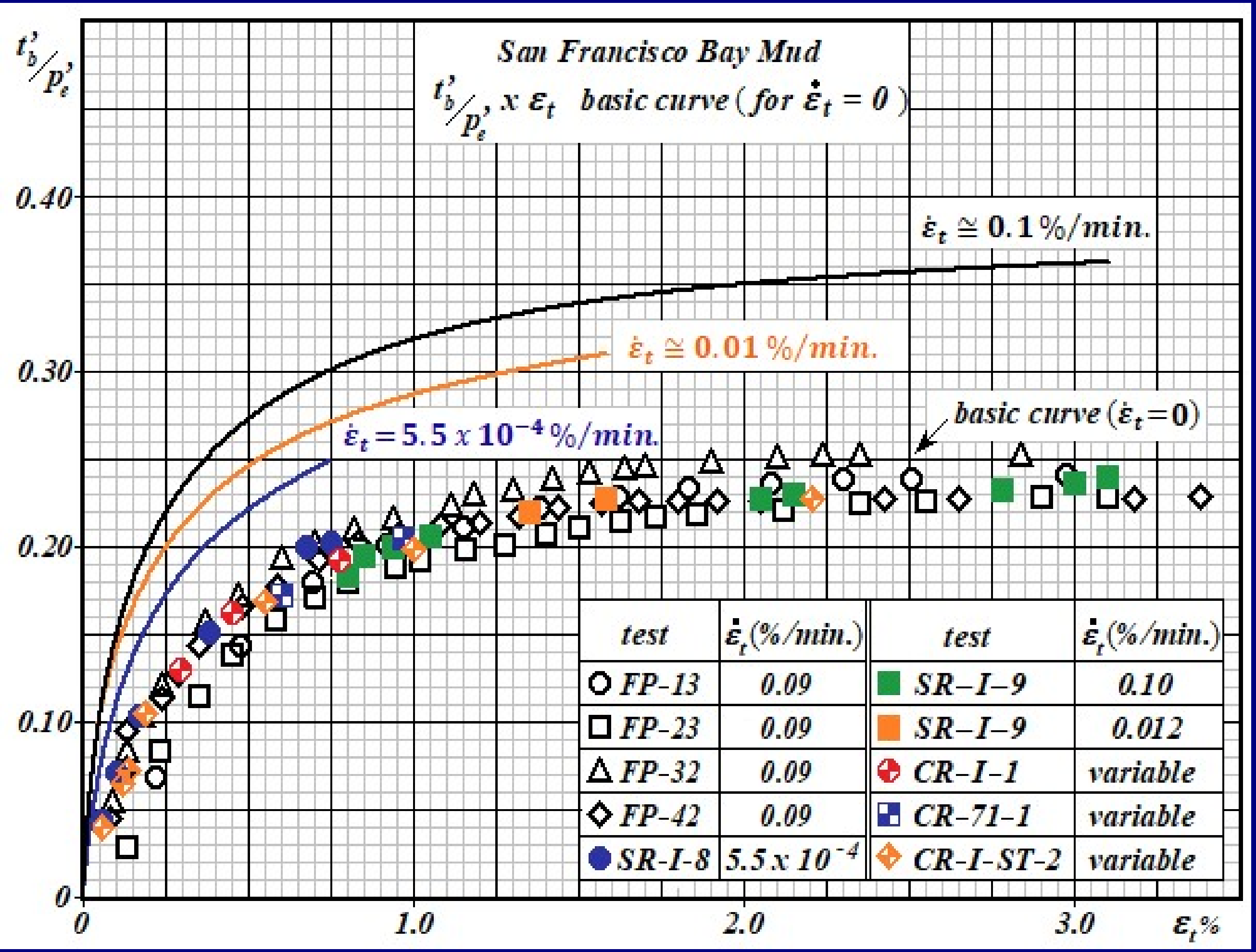
$$\frac{s'_b}{p'_e} = \left(\frac{s'}{p'_e} - \frac{t'}{p'_e}\right) + \frac{\left(\dfrac{t'}{p'_e} - \dfrac{\mathbb{V}}{p'_e}\right)^2}{\left(\dfrac{s'}{p'_e} + \dfrac{t'}{p'_e}\right)}\left[1 + \frac{\sqrt{\dfrac{s'^2}{p'^2_e} + \dfrac{\mathbb{V}^2}{p'^2_e} - \dfrac{2\mathbb{V}t'}{p'^2_e}}}{\left(\dfrac{t'}{p'_e} - \dfrac{\mathbb{V}}{p'_e}\right)}\right]$$

$$tan\ \phi'_{emob} = \frac{\left(\dfrac{t'}{p'_e} - \dfrac{\mathbb{V}}{p'_e}\right)}{\sqrt{\dfrac{s'^2}{p'^2_e} - \dfrac{t'^2}{p'^2_e}}}$$

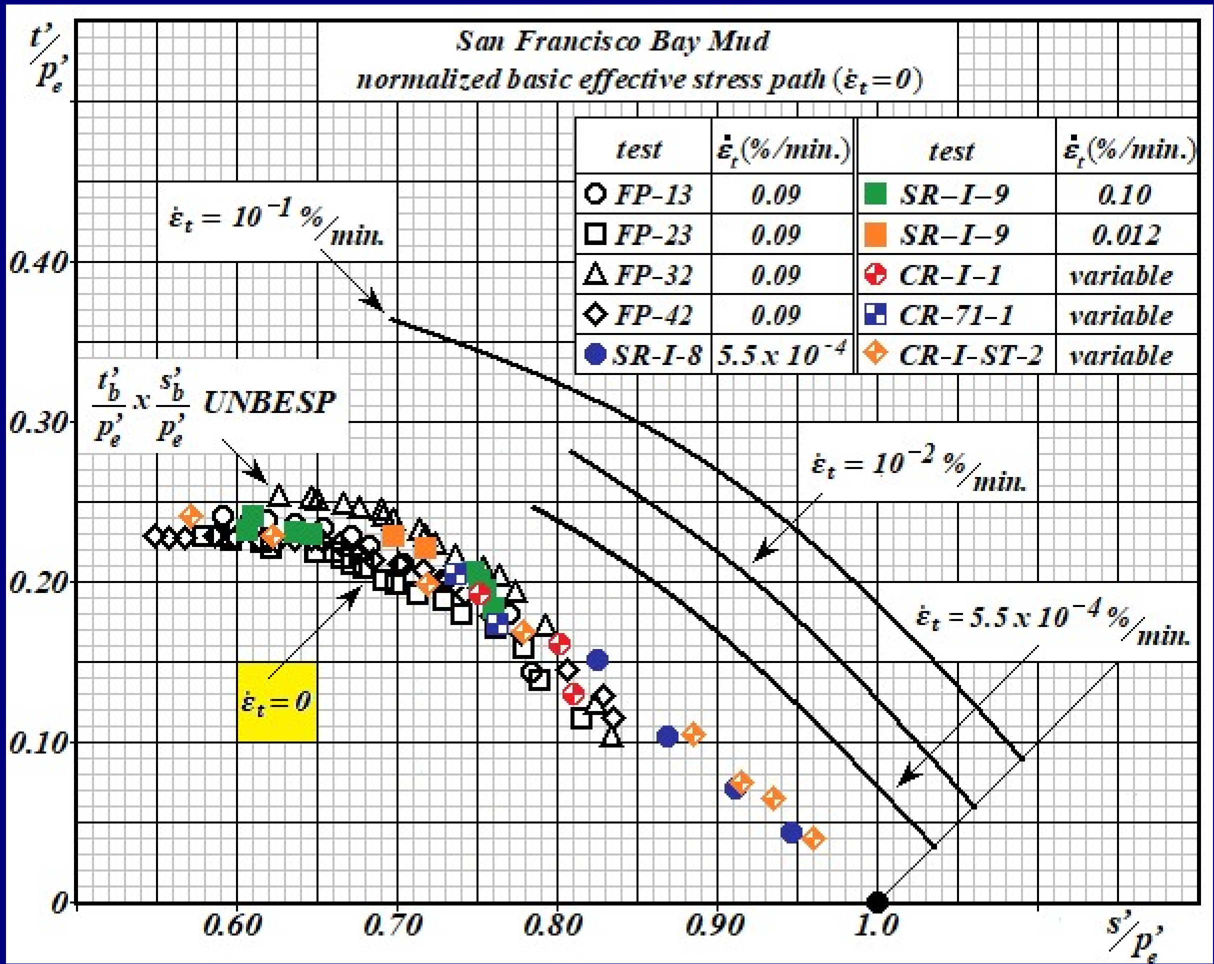
## $\Delta u / p'_e \; x \; \varepsilon_t$ unique curve.



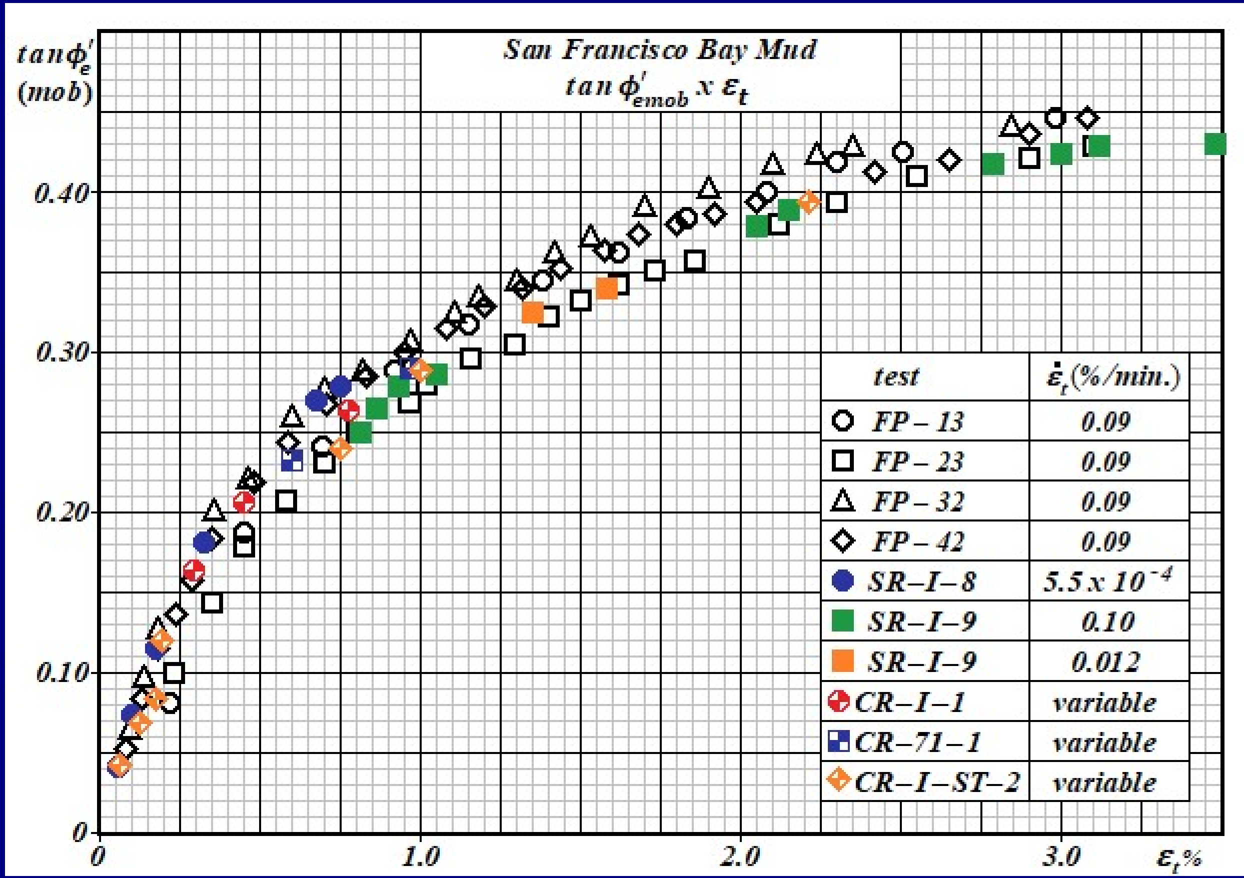
## *Unique basic curve $t'_b/p'_e \; x \; \varepsilon_t$*



## *Unique normalized basic effective stress path* $(s'_b/p'_e, t'_b/p'_e)$ *(UNBESP)*



# *Unique curve* $\tan \phi'_{emob} \, x \, \varepsilon_t$.



## Special undrained tests – undrained creep and stress relaxation tests

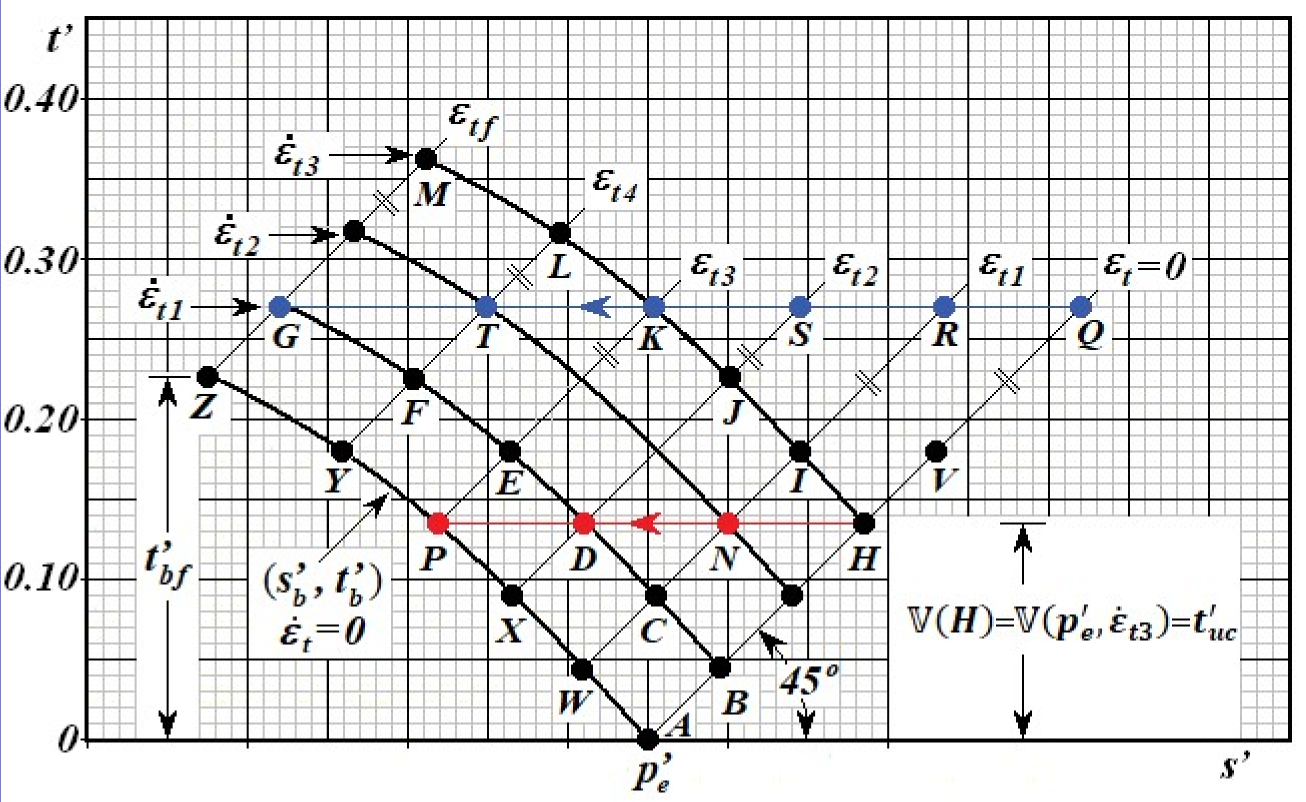
An undrained creep test is defined as a test where the state of total stress in a soil specimen is kept constant with time and the state of strain, the pore-pressure and the strain rate are observed along time under undrained conditions.

Creep tests presented in this article (*) are of the type $\overline{\boldsymbol{CIUCL}}$, i.e.: the specimen is consolidated to an isotropic stress $\boldsymbol{p'_e}$ and then subjected to a constant deviator stress $(\boldsymbol{\sigma_a} - \boldsymbol{\sigma_r})$. The total axial (vertical) stress is increased from $\boldsymbol{p'_e}$ to $\boldsymbol{\sigma_a}$ while the total radial (horizontal) stress $(\boldsymbol{\sigma_r})$ is kept constant and equal to $\boldsymbol{p'_e}$.

(*) a step creep test is a creep test where the deviator stresses is increased in steps keeping $\boldsymbol{\sigma_r} = \boldsymbol{p'_e}$ and increasing $\boldsymbol{\sigma_a}$ at the end of fixed time interval.

A stress relaxation test is actually a stage of a $\overline{\boldsymbol{CIUCL}}$ test during which the load frame motor is switched off and the pore-pressure and the deviator stress is observed under undrained condition.

The undrained creep




Undrained creep mechanism ?

Transference of viscous to friction resistance with time under constant deviator stress.

- Immediately after applying $\boldsymbol{t'(H) = t'_{uc} = constant}$, $\boldsymbol{ESP}$ will be at point $\boldsymbol{H}$ with $\boldsymbol{\varepsilon_t = 0}$, $\boldsymbol{\dot{\varepsilon}_t = \dot{\varepsilon}_{t3}}$ and zero mobilized friction resistance (all mobilized resistance is due to viscosity).

- After a given elapsed time, $\boldsymbol{ESP}$ will be at $\boldsymbol{N}$ with $\boldsymbol{\varepsilon_t = \varepsilon_{t1}}$, $\boldsymbol{\dot{\varepsilon}_t = \dot{\varepsilon}_{t2} < \dot{\varepsilon}_{t3}}$. Part of the viscous resistance was transferred to friction resistance.

- At $\boldsymbol{P}$, $\boldsymbol{\varepsilon_t = \varepsilon_{t3}}$, $\boldsymbol{\dot{\varepsilon}_t = 0}$ . All viscous resistance was transferred to friction. Creep comes to an end.

Undraned creep → analogous to one-dimensional consolidation.

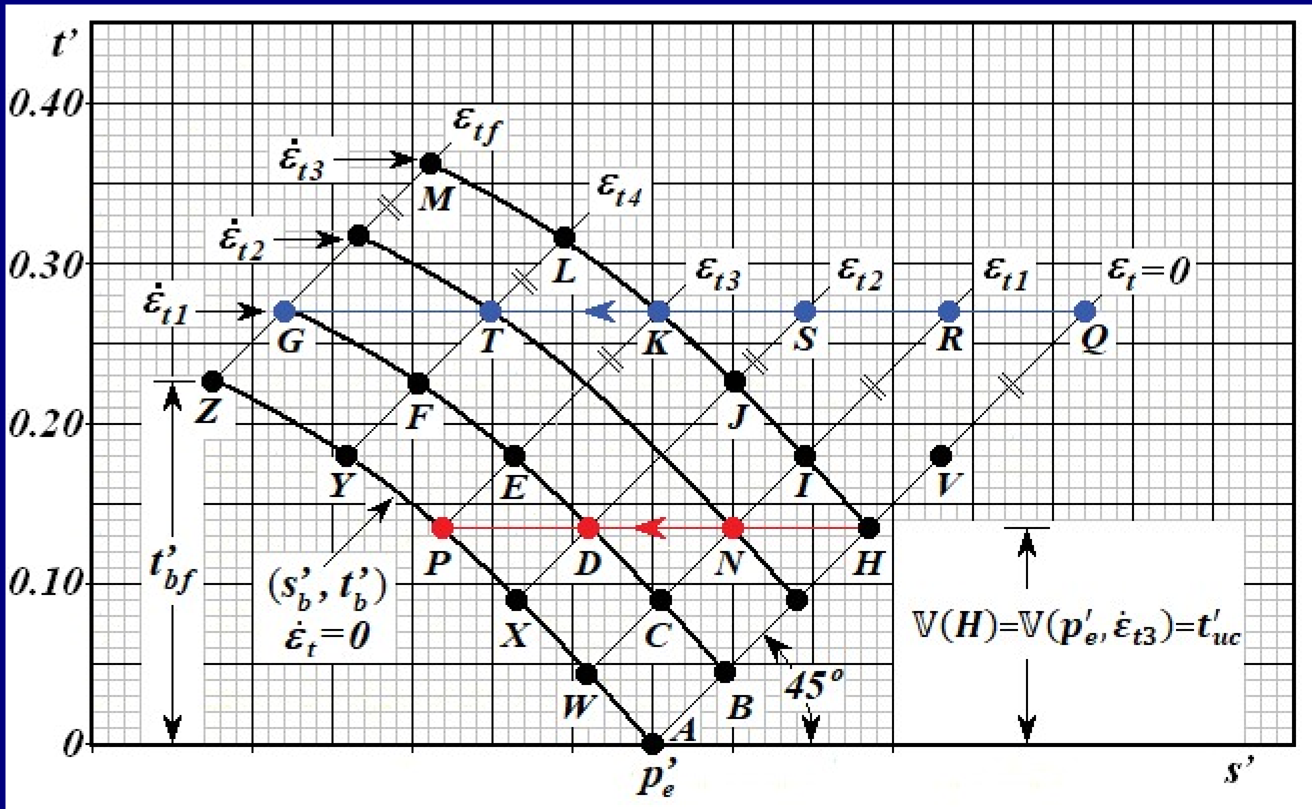
During one-dimensional consolidation the vertical total stress is kept constant and there is transference of the dissipated excess pore-pressure to the vertical effective stress causing soil deformation. During undrained creep there is transference of the viscous resistance to friction resistance. As $\boldsymbol{\eta}(\boldsymbol{e}) = constant$ and $\mathbb{V} = \boldsymbol{\eta}(\boldsymbol{e})\,\boldsymbol{f}(\dot{\boldsymbol{\gamma}})$, as $\mathbb{V}$ decreases with time, $\boldsymbol{f}(\dot{\boldsymbol{\gamma}})$ must decrease making $\dot{\boldsymbol{\gamma}}$ to decrease with time.

There is a special feature that makes the analogy between one-dimensional consolidation and undrained creep to be not a perfect one.

↓

Let's examine the undrained creep carried out under $\boldsymbol{t}' = \boldsymbol{t}'_{uc}$ along the effective stress path $\boldsymbol{QRSKTG}$ .

Undrained creep along the effective stress path $\boldsymbol{QRSKTG}$.




At Point G

$$\varepsilon_t = \varepsilon_{tf}$$

$$\phi'_{emob} = \phi'_e$$

$$\dot{\varepsilon}_t = \dot{\varepsilon}_{t1}$$

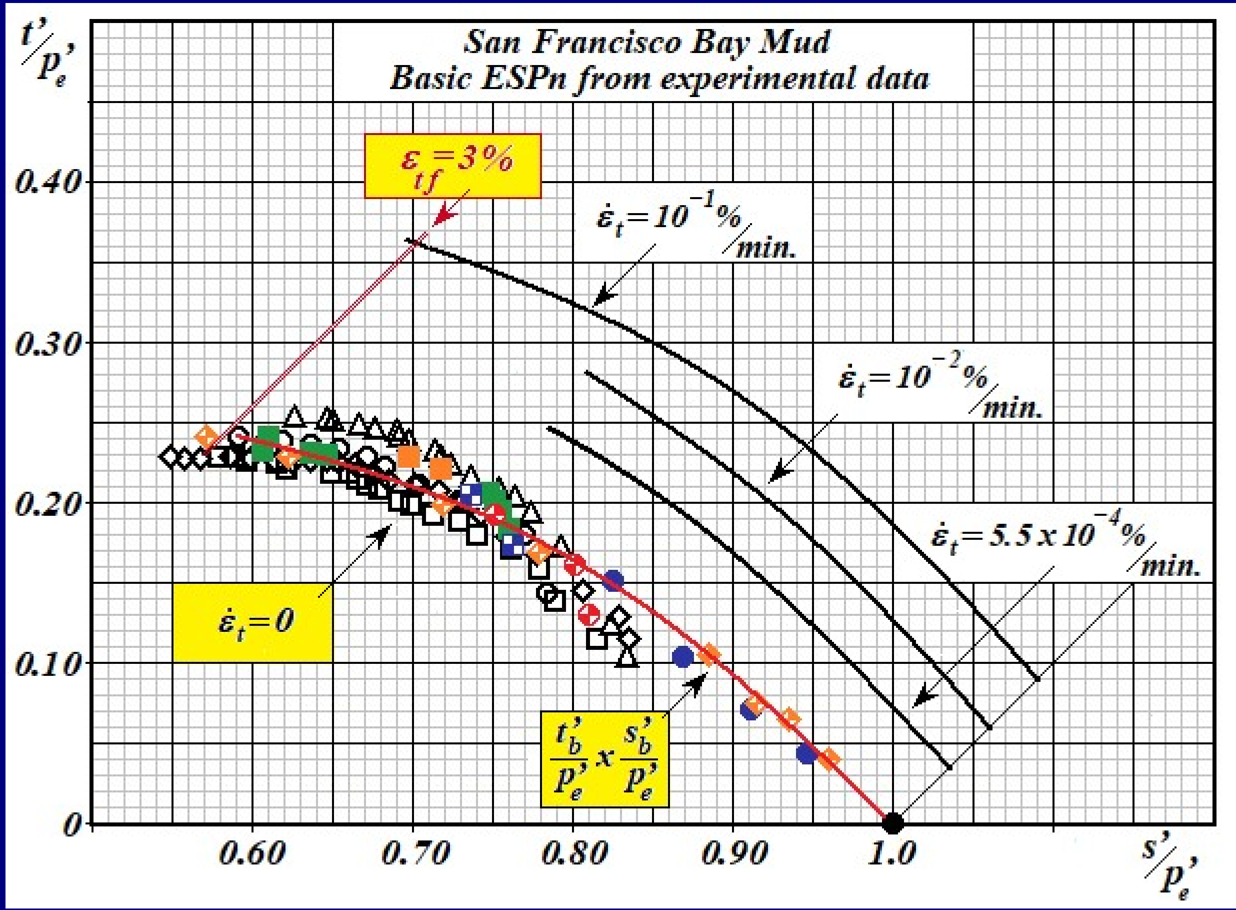
Friction resistance is fully mobilized and cannot increase.

It is necessary to keep an active viscous resistance to satisfy equilibrium.

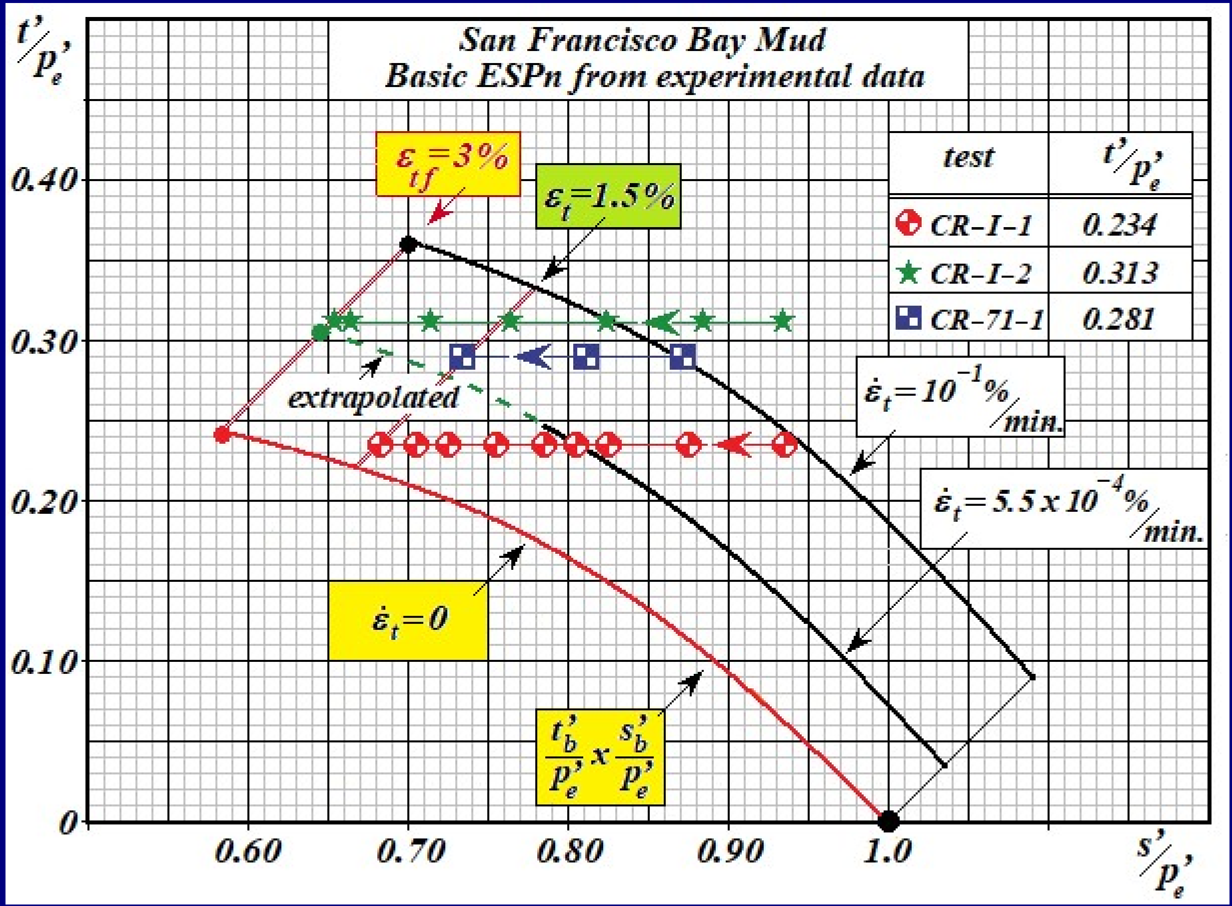
To keep an active viscous resistance it is necessary to keep a strain rate indefinitely ⇒ creep failure.

Undrained creep tests carried out on normally consolidated samples of San Francisco Bay Mud

Basic ***ESP*** determined from experimental data.



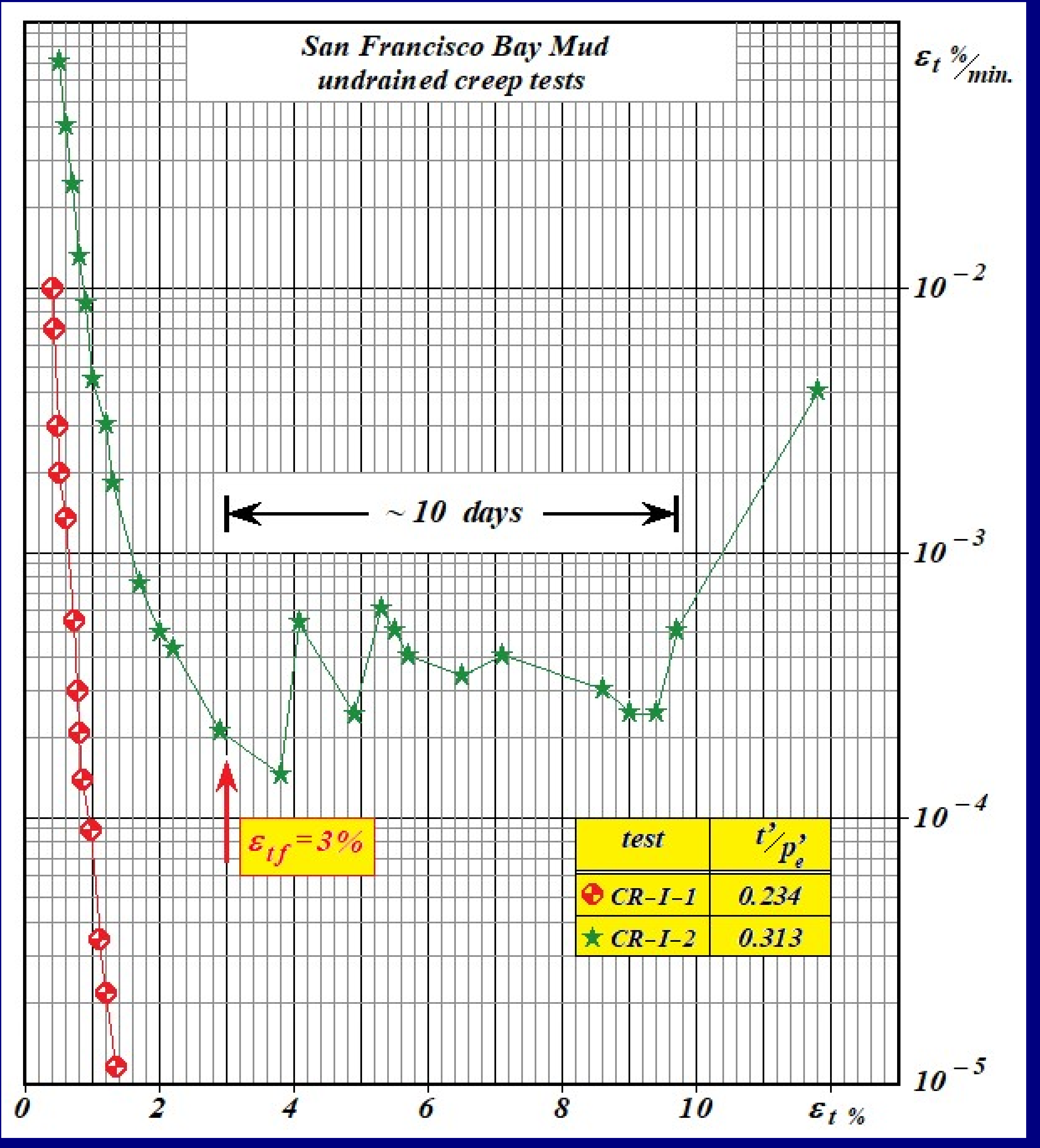
Undrained creep tests on normally consolidated samples of San Francisco Bay Mud (data from Lacerda, 1976)



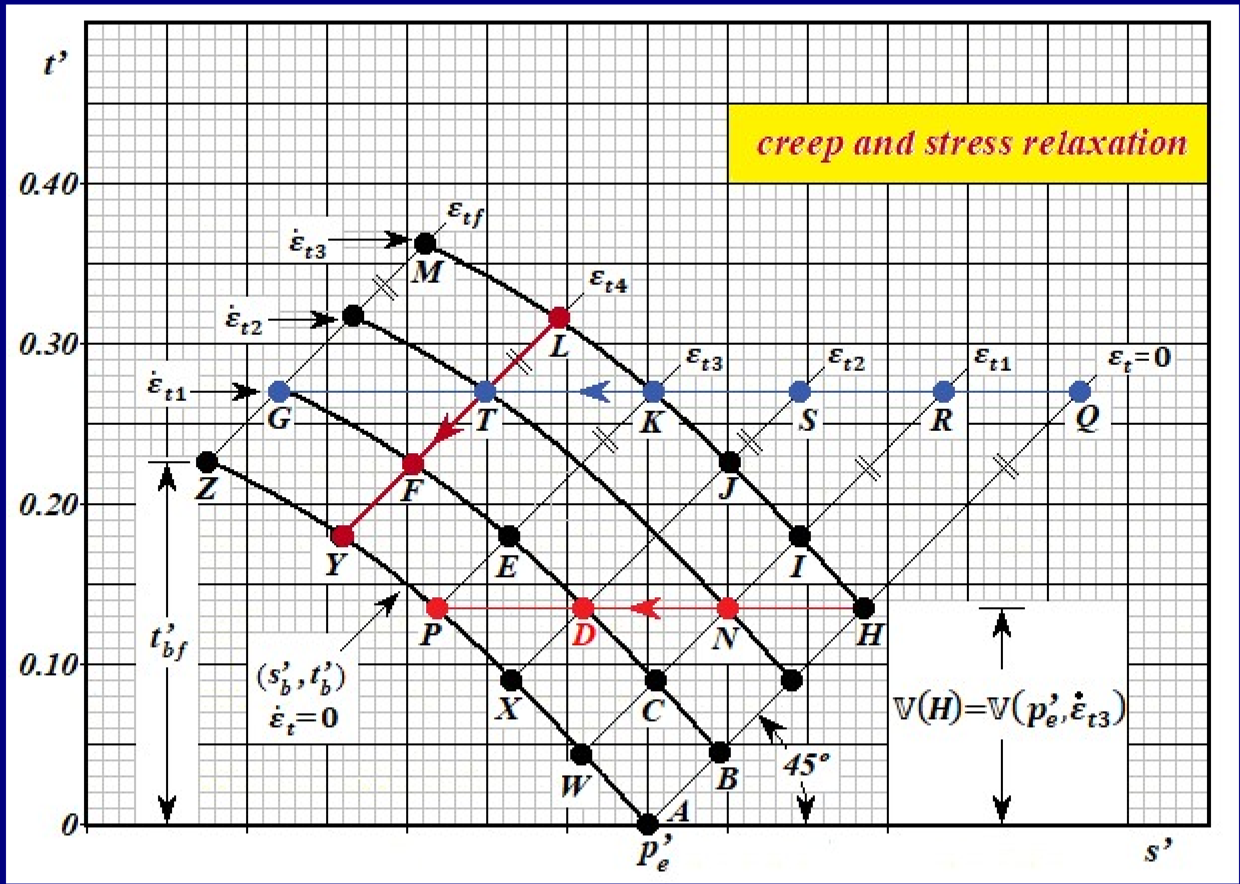

**CR – I – 1** ➡ ***creep should come to an end and $\varepsilon_t$ should tend $\cong 3\%$.***

**CR – I – 2** ➡ ***creep failure expected to occur for $\varepsilon_t \cong 3\%$ with $\dot{\varepsilon}_t \cong 5.5\ x\ 10^{-4}\ \%/min.$***

**CR – 71 – 1** ➡ ***test interrupted.***


San Francisco Bay Mud
undrained creep tests
~10 days
$\varepsilon_{tf} = 3\%$
test
$t'/p'_e$
CR-I-1
0.234
CR-I-2
0.313
$\dot{\varepsilon}_t$ %/min.
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
0
2
4
6
8
10
$\varepsilon_t$ %

## Creep and stress relaxation



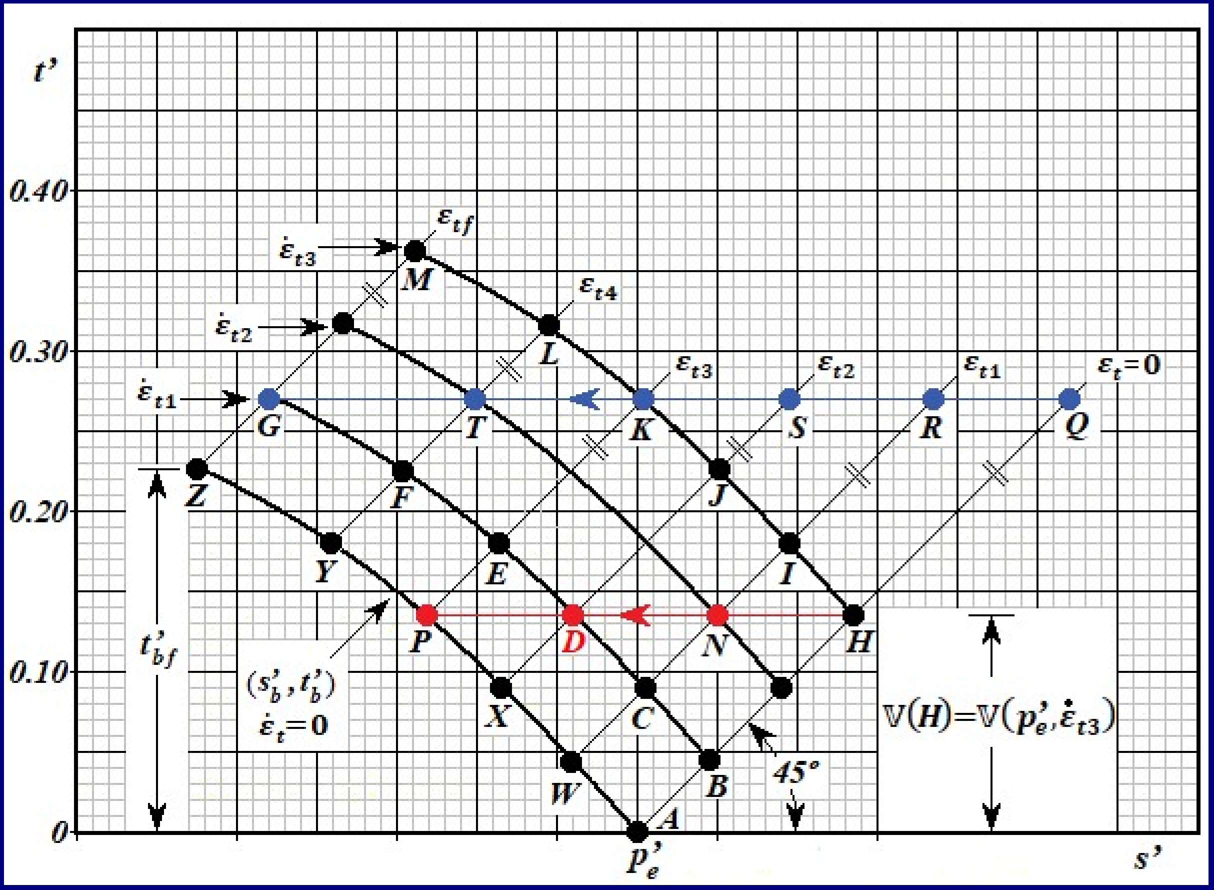

creep: ***HNDP***, ***QRSKTG***

stress relaxation: ***LTFY***

"creep" and strees relaxation ***ESPs*** crossing different Roscoe surfaces each one associated to a different strain rate.

*Taylor – Bjerrum generalized law*

*"A plastic soil subjected to a state of stress where the shear stresses are internally resisted by viscosity and friction will tend with time to get rid of shear resistance of viscous origin transfering them to shear resistance of frictional nature only."*



## Conclusions

- There are examples showing the principle of effective stress is not of general validity. These examples are usually found among phenomena involving strain rate effects and time effects like creep and stress relaxation.

- The word "***cohesion***" is used with different meanings that bring confusion and misunderstanding.

- The word "***cohesion***" as defined by Coulomb, is given by the the intersection of the strength envelope with the shear stress axis and implies a tensile strength. This tensile strength exists only when there is cementation between the soil grains.

- Terzaghi & Frölich (1936) use explicitly the word ***"viscosity"*** to refer to a component of shear strength existing in plastic soils that has its origin in the interaction of adsorbed viscous water surrounding clayey soil particles.

- The word ***"cohesion"*** is usually used in soil mechanics to describe a ***"sticky earthy material which is soft to the touch when moist".*** An earth material which show such characteristics is called a ***"cohesive soil"***. In the author's opinion, to avoid misunderstanding, these materials would be more properly called plastic soils rather than being named cohesive soils. The word ***"cohesive"*** brought conceptual confusion to soil mechanics and geotechnical engineering.

- The expression ***"plastic soil"*** should be used, for practical purposes and for the sake of conceptual clearness, to all soils that present liquid and plastic limit.

- Normally consolidated clays whose strength envelopes pass through the origin of a $\tau\, x\, \sigma'$ plot, do have plasticity but do not have cohesion (as defined by Coulomb). When saturated and subjected to undrained tests with the same void ratio, they present higher strengths when tested with higher strain rates. This feature suggest that what is called "cohesive resistance" should be more properly called "viscous resistance".

- It is the viscosity concept that comes from the action of adsorbed water which is behind the coefficient $\mu$ used by Bjerrum (1973) to “correct” the undrained shear strength measured in the vane shear test. The input variable used by Bjerrum (1973) to obtain the $\mu$ value is the plasticity index. The higher the plasticity index the lower the value of $\mu$ . This indicates the more plastic the soil the greater the influence of strain rate (or viscous resistance) on its undrained shear strength measurement.

- A model of behaviour was presented adding to the ***PES*** an equation assuming that in plastic solis the applied shear stresses are internally resisted by the sum of two components: one of friction nature and one of viscous nature.

- The introduction of the viscous component in shearing resistance allows to extend the ***PES*** creating a failure criterion taking into account the strain rate effect in plastic soils. This extension of the ***PES*** explains phenomena like creep and stress relaxation as natural consequences of it.

- The viscous component of the shear stress depends on the shear strain rate and on the void ratio. The state of mobilized viscosity is represented by the viscosity ellipse whose ordinates give the viscous component of the shear stress.

- The friction component of the shear stress depends on the shear strain (or the distortion) and on the normal effective stress. The state of mobilized friction is represented by the friction ellipse.

- The sum of the viscosity and friction ellipses give the Mohr circle. Both ellipses can only exist together because only the Mohr circle satisfy static equilibrium .

- Despite of shear stress is made up of a viscous and a friction component, failure is governed by friction mobilization. In a normally consolidated plastic soil failure occurs when the friction ellipse becomes tangent to the shear strength envelope, a straight line passing through the origin with slope $\phi'_e$ the Hvorslev true angle of friction.

- Except for the viscosity and friction ellipses, used to reorganize old ideas and concepts found in classic texts of soil mechanics (e.g. Terzaghi and Frölich (1936), Hvorslev (1937), Terzaghi (1938), Taylor (1942), Taylor (1948), Gibson (1953) and Bjerrum (1973)), there is nothing new in the posed approach making the following de Mello's thought somewhat prophetic:

***"We professionals beg less rapid novelties, more renewed reviewing of what is already there"***

***(Prof. Victor de Mello)***

Thank you very much !