CARRYING CAPACITY OF FOUNDATIONS ON OR IN GROUND: DIFFERENT PERSPECTIVES

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"The important thing in science is not so much to obtain new facts, as to discover new ways of thinking about them" –*Victor de Mello*

ABSTRACT: Conventional approaches for the estimation of ultimate capacities of shallow and deep foundations use limit equilibrium methods and are based only on soil strength parameters apart from the geometry of the foundation element. Ground/soil being a much more complex material than strong and very stiff metal, such as steel, for which the original theories have been developed, requires the consideration of stiffness as well as strength parameters for the estimation of ultimate loads. The paper summarises approaches for the estimation of carrying capacity of foundations on or in ground based on (i) strength as originally proposed by Terzaghi and subsequent developments in the same genre, (ii) strength and stiffness following Gibson, Vesic, etc., (iii) stiffness alone considering leaning instability as a failure mode, and (iv) in-situ test parameters, such as SPT *N* and CPT q_c. The results for bearing capacity of shallow foundations, ultimate axial and lateral capacities of piles, and leaning instability of tall structures, show dependence on the stiffness of the ground apart from its strength. The contributions of the relative stiffness of the ground on the ultimate bearing stresses, capacities or loads are quantified.

1. INTRODUCTION

All Civil Engineering structures need to be founded on or in the ground. Their basic design involves consideration of both stability and serviceability in the form of estimation of ultimate, safe and allowable bearing capacities of foundations. Terzaghi's theory for the ultimate bearing capacity of shallow foundations, which is based on Prandtl's theory originally developed for punching of metals, has been extensively developed and modified in current geotechnical practice. This theory has three fundamental assumptions which model the problem in a very simplistic manner and far from the real response of the ground due to the loads from the structure.

The system consists of (i) the superstructure that is above the ground and (ii) the ground below with a structural element termed as foundation that transfers the load from the superstructure to the ground in a suitable and permissible manner. In all our analyses, the effect of the structure, particularly its aspect ratio, i.e., height to width, which has a major bearing on the performance of the structure, is often neglected.

The second most important aspect is that Prandtl's theory is based on rigid-plastic stress–strain response which is fairly true for strong and stiff material, such as steel, but most inappropriate for ground which is weak, soft and highly compressible. Rigid-plastic behaviour implies infinitely small strain or deformation before failure; however, soils typically attain peak strength at strains of the order of 5 to 20%.

The estimation of bearing capacity of foundations is one of the foundations of geotechnical engineering. The classic work of Terzaghi (1943), based on Prandtl's solution for metals, is the starting point for this topic. The initial approaches for the study of soil as an engineering material had to rely profusely on the studies of other engineering materials, especially of metals. The stiffness of these engineering materials is so high that strains mobilized at or near failure are small enough to be ignored. Thus, rigid plasticity theory, which neglects deformations or strains, is appropriate to study the states of failure only in such materials. Hence, the failure state is examined considering only the equations of equilibrium and the failure or yield criterion.

2. BEARING CAPACITY OF SHALLOW FOUNDATIONS (Based on Soil Strength Alone)

Prandtl's theory (Fig. 1), originally developed for metals with compressive and tensile strengths of nearly the same magnitude, i.e., with friction angle equal to zero, is the starting point for the estimation of bearing capacity of shallow foundations. Terzaghi (1943) modified the same and proposed his famous theory for a strip footing embedded in a cohesive–frictional soil (Fig. 2). The slip mechanism consists of an active rigid-elastic wedge defined by the angle of shearing resistance ϕ, a fan region of continuous plastic deformation (distortion+rotation), and passive wedges defined by the angle ($\pi/4 - \phi/2$), both with respect to the horizontal. Prandtl's solution modified for c–ϕ soils with the active wedge defined by ($\pi/4 + \phi/2$), instead of ϕ, is adopted as the basis for the estimation of ultimate bearing capacity q_u of shallow foundations, as

$$q_u = cN_c s_c + qN_q s_q + 0.5\gamma B N_\gamma s_\gamma \tag{1}$$

where N_c, N_q and N_γ are bearing capacity factors while s_c, s_q and s_γ are shape factors. N_q and N_γ increase significantly with ϕ leading to extremely high bearing capacities. It was soon realized that real soils do not fail only in 'general shear failure' and a new failure mode, termed 'local shear failure', was identified as a possible alternative. The ultimate bearing capacity of footings based on local shear failure is estimated empirically using Eq. (1) but with c and $\tan\phi$ reduced to two-thirds their corresponding values (Terzaghi and Peck 1967). Vesic (1973) extended this concept and identified a third failure mode, namely 'punching shear failure', occurring in loose soils at shallow depths and at depth in case of dense soils. Fig. 3 classifies the three failure modes in terms of both the relative density of sand and the relative depth of foundation.

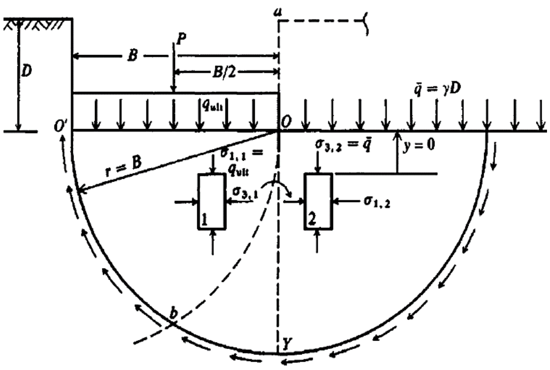

Fig. 1 Bearing capacity failure in clay ($\phi_u = 0$) (Prandtl's theory)

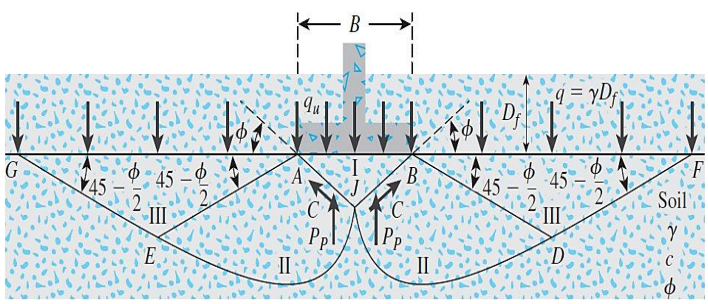

Fig. 2 General shear failure mechanism in c–ϕ soil (Terzaghi 1943; adapted from Das 2011)

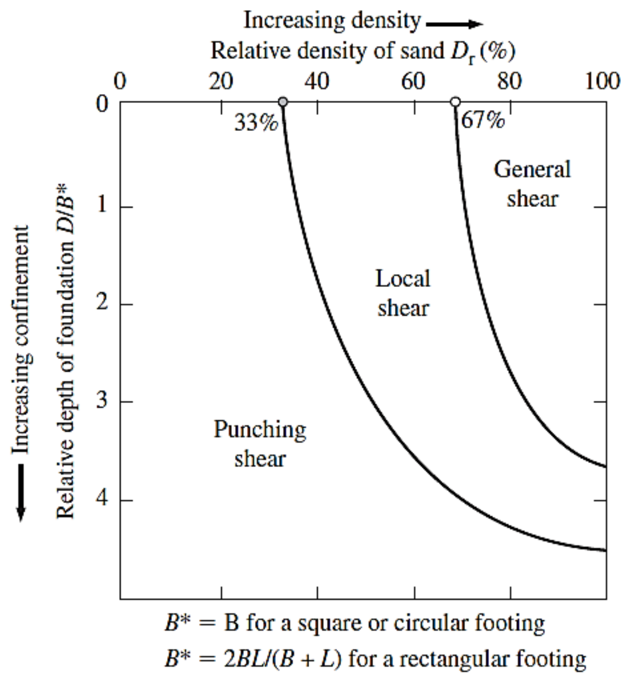

Fig. 3 Modes of failure of shallow foundations in sand (Vesic 1973; adapted from Salgado 2008)

3. CAVITY EXPANSION THEORY (Based on Strength and Compressibility of Soil)

Gibson and Anderson (1961) presented a plasticity solution for the limit pressure p_l for the expansion of a cylindrical cavity in undrained soft soil. Menard (1957) obtained the limit pressure of a cylindrical cavity in cohesive soil (ϕ_u = 0) in the form of an expression very similar to that for the bearing capacity of a footing as

$$p_l = c_u N_c^* + \sigma_h \tag{2}$$

where N_c^* = 1+ln(G/c_u), G and c_u are the shear modulus and the undrained strength of soft soil, respectively, and σ_h is the total horizontal stress in the ground. The factor N_c^*, which for the first time incorporates the relative stiffness of the ground with respect to its strength, increases with the rigidity index $I_r = G/c_u$ (Fig. 4). Thus, the limit cavity pressure depends on the shear stiffness apart from the undrained strength of the soil. Similar expressions have been derived by Vesic (1972) for estimating the limit pressure for the expansion of a cylindrical cavity in c–ϕ soil as

$$p_l = cF_c + q'F_q \tag{3}$$

$$F_q = (1+\sin\phi)(I_{rr}\sec\phi)^{\sin\phi/(1+\sin\phi)} \tag{4}$$

$$F_c = (F_q - 1)\cot\phi \tag{5}$$

$$I_{rr} = \frac{G/(c+q'\tan\phi)}{1+(G/(c+q'\tan\phi))\varepsilon_v^p \sec\phi} \tag{6}$$

where F_c and F_q are cylindrical cavity expansion factors for cohesion and surcharge, respectively, I_{rr} is the reduced rigidity index, q' is the isotropic effective stress in the soil mass, and ε_v^p is the average volumetric strain in the plastic zone around the cylindrical cavity.

Figure 5 depicts the variation of R_q, the ratio of the limit cavity pressure of compressible ground to that of incompressible ground ($G \rightarrow \infty$), with the normalized stiffness $G/\gamma D$ and the angle of shearing resistance ϕ, for $\varepsilon_v^p = 0.3\%$ and $c/\gamma D = 0.5$, where D is the depth of the footing below the ground surface. It is observed that R_q increases significantly at low values of $G/\gamma D$ for ϕ values ranging from 20° to 45° but is nearly independent of ϕ for a given $G/\gamma D$. Fig. 6 shows that the effect of $c/\gamma D$ on R_q is remarkable for values of $G/\gamma D$ less than 15 at $\varepsilon_v^p = 0.3\%$ and $\phi = 30°$. Fig. 7 illustrates the variation of R_q with the reduced rigidity index I_{rr} and the angle of shear resistance ϕ for $\varepsilon_v^p = 0.3\%$ and $c/\gamma D = 0.5$. The values of R_q increase with I_{rr} for a given ϕ value and increase with ϕ for a given value of I_{rr}.

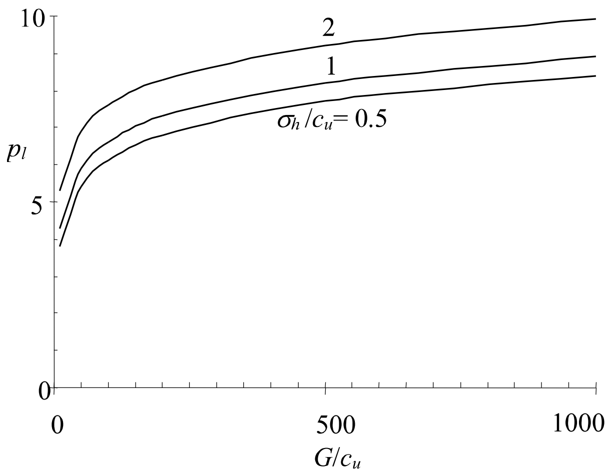

Fig. 4 Variation of limit pressure with shear stiffness and total horizontal stress

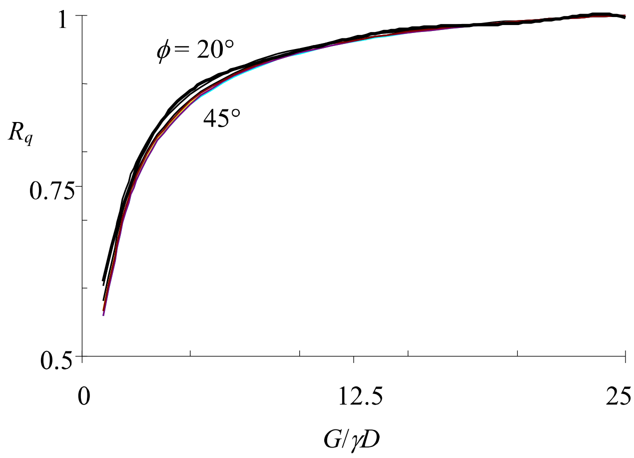

Fig. 5 Variation of R_q with $G/\gamma D$ and ϕ for $\varepsilon_v^p = 0.3\%$ and $c/\gamma D = 0.5$

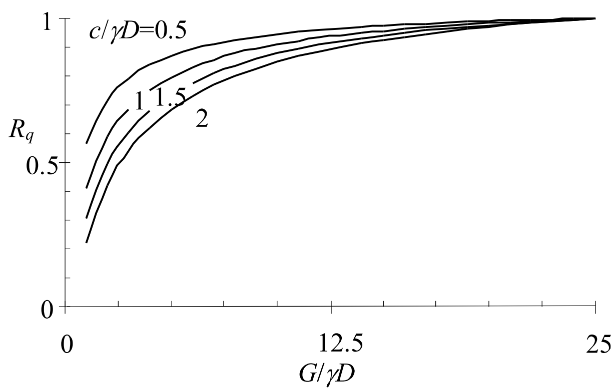

Fig. 6 Variation of R_q with $G/\gamma D$ and $c/\gamma D$ for $\varepsilon_v^p = 0.3\%$ and $\phi = 30°$

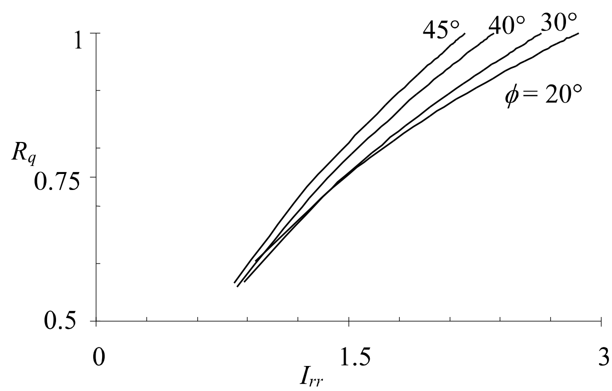

Fig. 7 Variation of R_q with I_{rr} and ϕ for $\varepsilon_v^p = 0.3\%$ and $c/\gamma D = 0.5$

4. BEARING CAPACITY OF SHALLOW FOUNDATIONS IN COMPRESSIBLE GROUND (Based on Cavity Expansion Theory)

Vesic (1973) proposed a general expression for the ultimate bearing capacity of shallow foundations accounting for the compressibility of the ground/soil as

$$q_u = cN_c F_{cs} F_{cd} F_{cc} + qN_q F_{qs} F_{qd} F_{qc} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma c} \tag{7}$$

where F_{cc}, F_{qc}, $F_{\gamma c}$ are compressibility factors, F_{cs}, F_{qs}, $F_{\gamma s}$ are shape factors, and F_{cd}, F_{qd}, $F_{\gamma d}$ are depth factors. The soil compressibility factors were derived by Vesic (1973) by analogy to the expansion of cavities. F_{cc}, F_{qc}, and $F_{\gamma c}$ are obtained in terms of the rigidity index I_r of the soil at a depth of approximately $B/2$ measured from the bottom of the foundation, as

$$I_r = \frac{G}{c + q' \tan\phi} \tag{8}$$

where q' is the effective overburden pressure at a depth of $D + B/2$ below the ground surface. The critical rigidity index $I_{r(cr)}$ for rigid-plastic condition is expressed as

$$I_{r(cr)} = 0.5\left\{\exp\left[\left(3.3 - 0.45\frac{B}{L}\right)\cot\left(45 - \frac{\phi}{2}\right)\right]\right\} \tag{9}$$

The compressibility factors, F_{cc}, F_{qc}, and $F_{\gamma c}$ are given as follows

(1) For $I_r < I_{r(cr)}$

$$F_{qc} = F_{\gamma c} = \exp\left\{\left(-4.4 + 0.6\frac{B}{L}\right)\tan\phi + \left[\frac{(3.07\sin\phi)(\log 2I_r)}{1 + \sin\phi}\right]\right\} \tag{10}$$

(a) For frictionless soils ($\phi = 0°$)

$$F_{cc} = 0.32 + 0.12\frac{B}{L} + 0.6\log I_r \tag{11}$$

(b) For cohesive–frictional soils ($\phi > 0°$)

$$F_{cc} = F_{qc} - \frac{1 - F_{qc}}{N_q \tan\phi} \tag{12}$$

(2) For $I_r \geq I_{r(cr)}$, $F_{cc} = F_{qc} = F_{\gamma c} = 1$ and correspond to rigid-plastic (general shear failure)

Equation (7) is normalized with γB as

$$\frac{q_u}{\gamma B}=\left(\frac{c}{\gamma B}\right)N_c F_{cs} F_{cd} F_{cc}+\left(\frac{D}{B}\right)N_q F_{qs} F_{qd} F_{qc}+0.5N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma c} \tag{13}$$

The ratio of the ultimate bearing capacity of the footing in compressible ground to that in incompressible ground, q_u/q_{ur}, is determined for B/L = 0 (strip), 0.2, 0.5, and 1 (square); D/B = 0.5, 1, 1.5, 2, 2.5 and 3; and $c/\gamma B$ = 0, 0.25, 0.5, 0.75 and 1. The values of the rigidity index I_r are varied from 1 to 500. It is observed that the bearing capacity ratio q_u/q_{ur} increases with I_r for a given ϕ at $B/L = 0$, $c/\gamma B = 0$ and $D/B = 1$ (Fig. 8). The limiting value of I_r beyond which q_u/q_{ur} remains constant at one, is equal to 55 for $\phi = 20°$ and 300 for $\phi = 35°$. General shear failure is expected for I_r values that are greater than these limiting values.

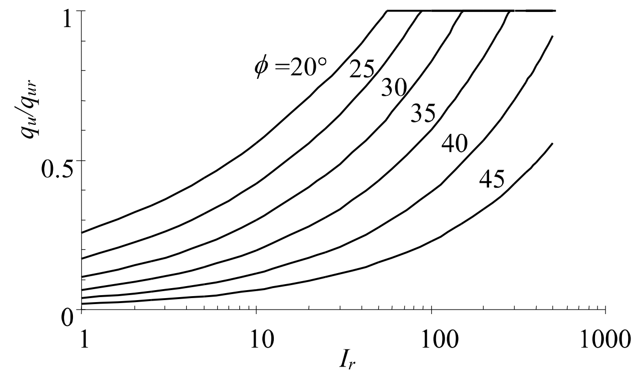

Fig. 8 Variation of q_u/q_{ur} with I_r and ϕ for $B/L = 0$, $c/\gamma B = 0$ and $D/B = 1$

Figures 9, 10 and 11 show the variations of q_u/q_{ur} with I_r and $c/\gamma B$ for $B/L = 0$, $D/B = 1$ and ϕ = 20°, 30° and 45°, respectively. The ratio q_u/q_{ur} is very small (of the order of 0.1 or less) for highly compressible soils, and increases gradually with I_r till it attains a value of 1 corresponding to general shear failure. The limiting value of I_r is found to be independent of $c/\gamma B$ for a given value of ϕ. Furthermore, q_u/q_{ur} is independent of $c/\gamma B$ for $\phi = 45°$. Similarly, the bearing capacity ratio q_u/q_{ur} is not very sensitive to the normalized depth of embedment of the footing D/B, especially at higher values of I_r (Fig. 12).

The limiting value of I_r increases with the angle of shearing resistance ϕ, as shown in Fig. 13, for $B/L = 1$, $D/B = 1$ and $c/\gamma B = 0.5$. The limiting value of I_r is 30 for $\phi = 20°$ and increases to 500 for $\phi = 45°$. The ratio q_u/q_{ur} is very sensitive to ϕ and decreases from about 0.6 for $\phi = 20°$ to about 0.1 for $\phi = 45°$ at $I_r = 10$. Fig. 13 thus highlights the significant effect of soil stiffness on the ultimate bearing capacity of shallow foundations.

The variations of the bearing capacity ratio q_u/q_{ur} with I_r for strip, square and rectangular footings are shown in Fig. 14. The limiting value of I_r deceases from about 150 for B/L = 0 (strip) to 70 for B/L = 1 (square), for $\phi = 30°$, $D/B = 1$ and $c/\gamma B = 0.5$. Similar variation of q_u/q_{ur} with I_r for different B/L ratios can be observed in Fig. 15 for $\phi = 45°$. While the trend in the variation of q_u/q_{ur} with I_r for $\phi = 45°$ is similar to that for $\phi = 30°$; however, the curves for $\phi = 45°$ do not indicate any limiting values of I_r. The difference in the values of q_u/q_{ur} as B/L increases from 0 (strip) to 1 (square) is equal to 0.45 for $I_r = 500$ but is less than 0.05 for $I_r < 10$.

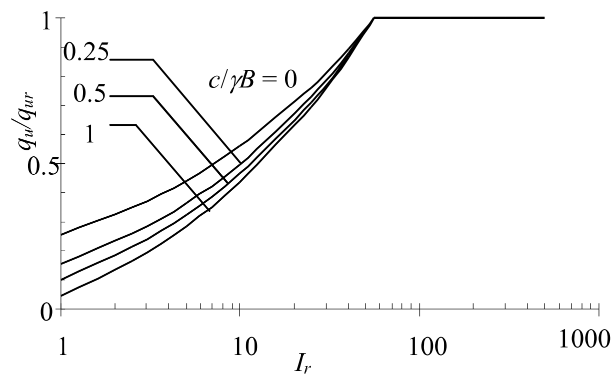

Fig. 9 Variation of q_u/q_{ur} with I_r and $c/\gamma B$ for $B/L = 0$, $\phi = 20°$ and $D/B = 1$

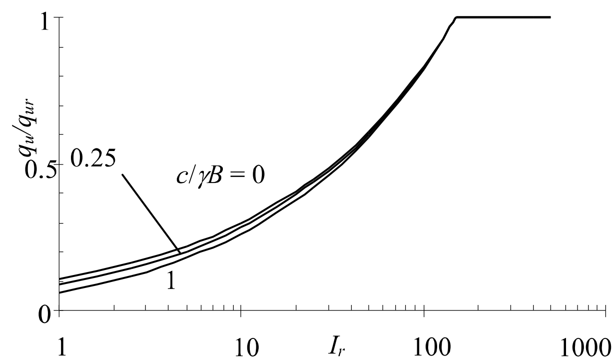

Fig. 10 Variation of q_u/q_{ur} with I_r and $c/\gamma B$ for $B/L = 0$, $\phi = 30°$ and $D/B = 1$

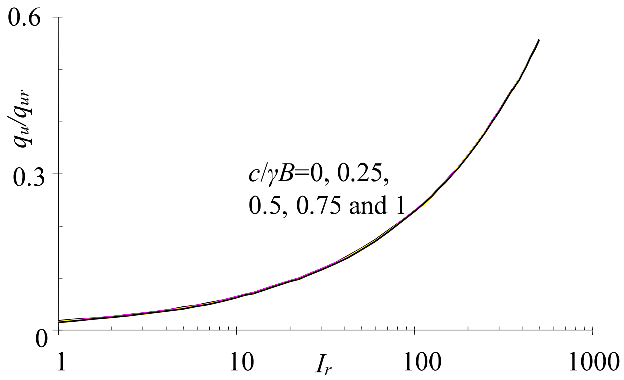

Fig. 11 Variation of q_u/q_{ur} with I_r and $c/\gamma B$ for $B/L = 0$, $\phi = 45°$ and $D/B = 1$

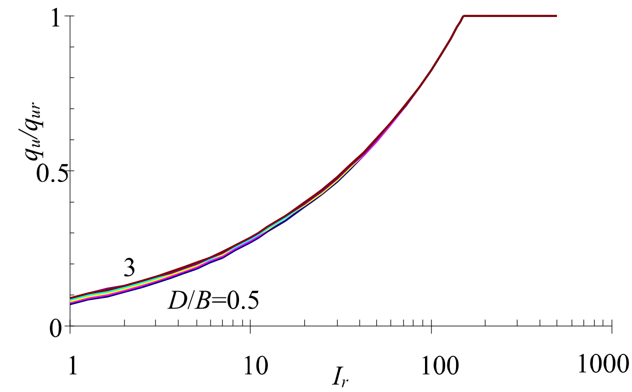

Fig. 12 Variation of q_u/q_{ur} with I_r and D/B for $B/L= 0$, $\phi = 30°$ and $c/\gamma B = 0.5$

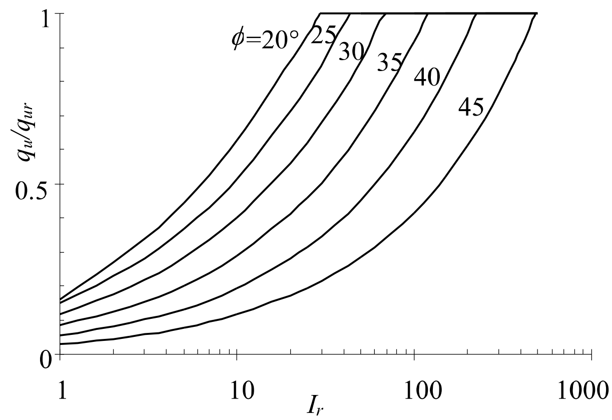

Fig. 13 Variation of q_u/q_{ur} with I_r and ϕ for $B/L = 1$, $D/B = 1$ and $c/\gamma B = 0.5$

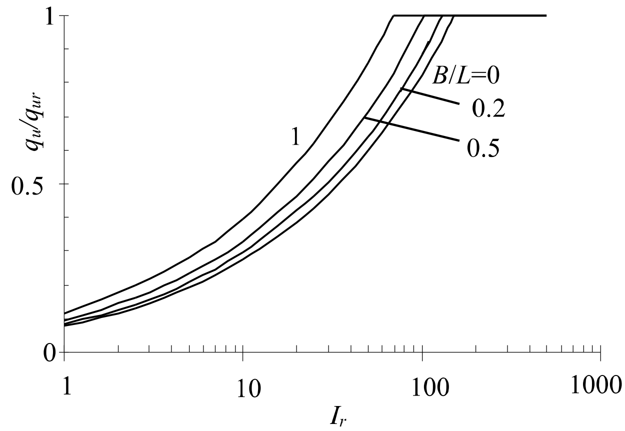

Fig. 14 Variation of q_u/q_{ur} with I_r and B/L for $D/B = 1$, $\phi = 30°$ and $c/\gamma B = 0.5$

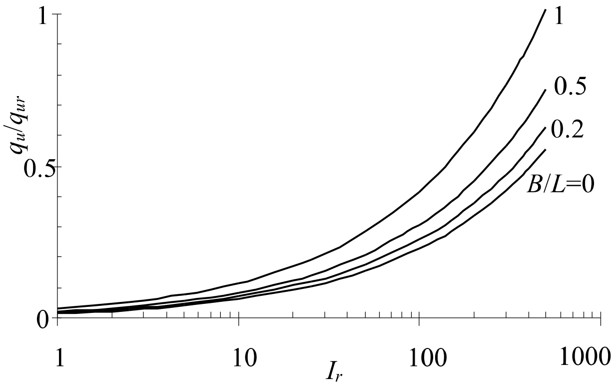

Fig. 15 Variation of q_u/q_{ur} with I_r and B/L for $D/B = 1$, $\phi = 45°$ and $c/\gamma B = 0.5$

5. ULTIMATE AXIAL POINT CAPACITY OF PILE FOUNDATIONS (Based on Cavity Expansion Theory)

Vesic (1977) proposed a method for estimating the pile point bearing resistance based on the theory of expansion of cavities. The proposed equation for ultimate point or end bearing resistance q_{pu} of the pile, is

$$q_{pu} = cN_c^* + \sigma'_m N_\sigma^* \tag{14}$$

$$\sigma'_m = \left(\frac{1+2K_0}{3}\right)q' \tag{15}$$

$$N_c^* = \frac{4}{3}\left(\ln I_{rr} + 1\right) + \frac{\pi}{2} + 1 \quad (\text{for } \phi = 0°) \tag{16}$$

where N_c^* and N_σ^* are bearing capacity factors with respect to cohesion and stress, respectively, σ'_m is the mean effective stress at the level of the pile base, and K_0 is the lateral earth pressure coefficient at-rest (equal to $1-\sin\phi$). According to Vesic's theory, $N_\sigma^* = f(I_{rr})$ where I_{rr} is the reduced rigidity index of soil defined as

$$I_{rr} = \frac{I_r}{1 + I_r \varepsilon_v^p} \tag{17}$$

where I_r is the rigidity index of soil (Eq. (8)) and ε_v^p is the average volumetric strain in the plastic zone below the pile point.

Figure 16 depicts the variation of q_{pu}/q_{pur}, the ratio of the ultimate point resistance of the pile in compressible ground to that in incompressible ground, with I_r and ϕ for $D/B = 10$ and $c/\gamma B = 0.5$. Here, D is the depth of the pile base below the ground surface and B is the pile diameter. It is observed that q_{pu}/q_{pur} decreases from about 0.25 to 0.1 as ϕ increases from 20° to 50° for $I_r = 10$. Thus, the effect of soil compressibility on the ultimate point resistance of the pile is greater for soils with higher angles of shearing resistance. The values of q_{pu}/q_{pur} converge at I_r = 500 for all ϕ values. Fig. 17 shows that the variation of q_{pu}/q_{pur} with I_r is independent of D/B in the range 10 to 100 for ϕ = 30° and $c/\gamma B$ = 0.5. Fig. 18 shows that $c/\gamma B$ has no effect on the variation of q_{pu}/q_{pur} with I_r for ϕ = 30° and D/B = 10. Fig. 19 illustrates the variation of q_{pu}/q_{pur} with I_r and ε_v^p for $c/\gamma B$ = 0.5, ϕ = 30° and D/B = 10. q_{pu}/q_{pur} increases with increase in I_r for a given volumetric strain; however, the value of the rigidity index decreases with increase in volumetric strain. The limiting value of I_r is 30 for ε_v^p = 3% and increases to 80 for ε_v^p = 1%.

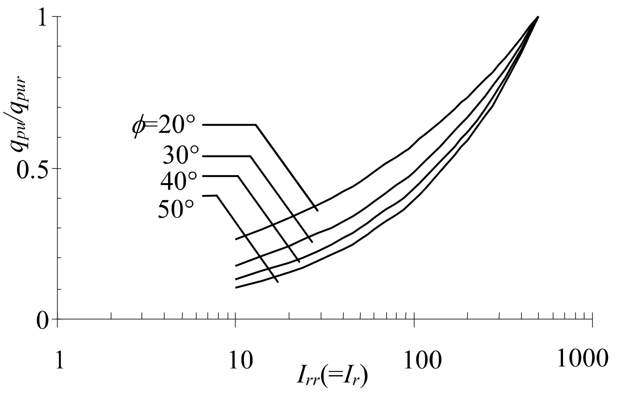

Fig. 16 Variation of q_{pu}/q_{pur} with I_r and ϕ for D/B = 10 and $c/\gamma B$ = 0.5

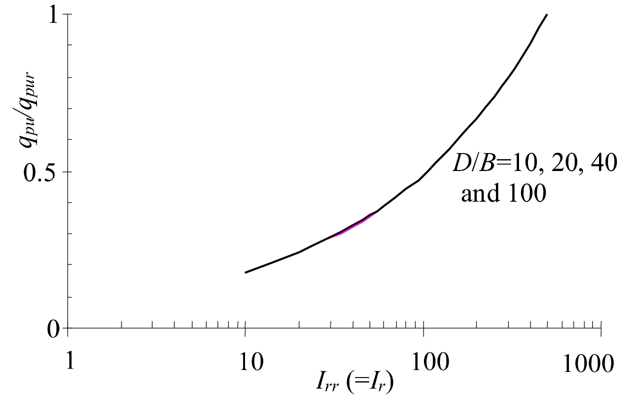

Fig. 17 Variation of q_{pu}/q_{pur} with I_r and D/B for ϕ = 30° and $c/\gamma B$ = 0.5

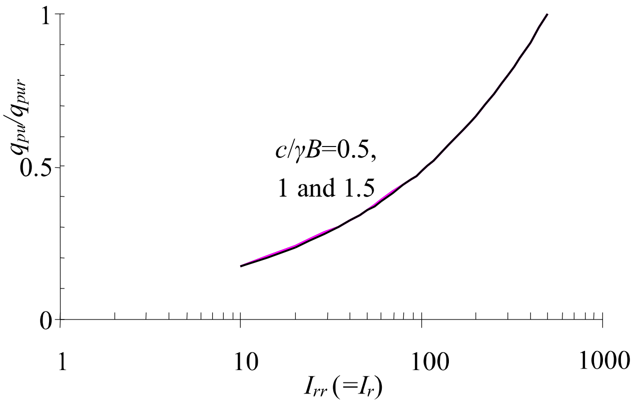

Fig. 18 Variation of q_{pu}/q_{pur} with I_r and $c/\gamma B$ for ϕ = 30° and D/B = 10

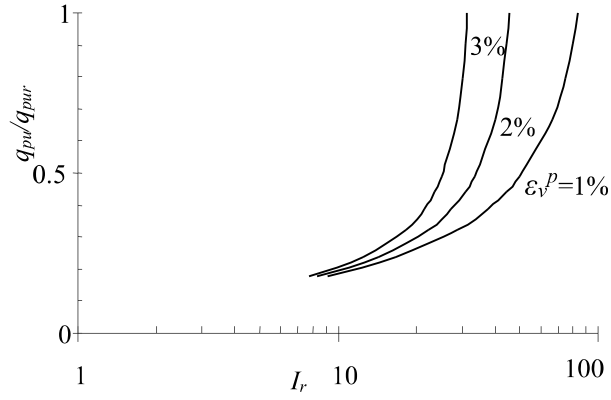

Fig. 19 Variation of q_{pu}/q_{pur} with I_r and ε_v^p for $c/\gamma B$ = 0.5, ϕ = 30° and D/B = 10

6. ULTIMATE LATERAL CAPACITY OF PILE FOUNDATIONS (Based on Soil Strength and Stiffness)

Padmavathi *et al.* (2007) proposed a method to estimate the behaviour of a laterally-loaded vertical rigid pile in cohesionless soil based on kinematics and non-linear subgrade reaction. A relative stiffness factor $\mu = k_sL/q_{max}$ (where k_s is the horizontal subgrade modulus of soil, L is the embedded length of the pile, and q_{max} is the ultimate lateral soil pressure) is introduced and the normalized ultimate lateral pile capacity, based on μ and L/d (where d is the pile diameter), is predicted. The ultimate lateral pile capacity is shown to depend on both the stiffness and the ultimate lateral pressure of the soil unlike in the theories of Broms (1964a, 1964b), Poulos (1971), Prasad and Chari (1999), and Zhang *et al.* (2005).

Figure 20 depicts the dependence of the normalized ultimate lateral pile capacity $H_u/K_p^2\gamma d^3$ on μ and L/d for the case of no moment at the ground point. H_u is the ultimate lateral capacity of the pile, K_p is the Rankine passive earth pressure coefficient, and γ is the moist unit weight of soil. It is observed that the normalized ultimate lateral pile capacity increases with μ for a given value of L/d. The effect of μ on the ultimate lateral pile capacity is negligible for very short piles ($L/d < 5$) but is significant for longer piles. The normalized ultimate lateral pile capacity increases from about 48 to 60 as μ increases from 5 to 2000 for $L/d = 24$.

Padmavathi *et al.* (2008) proposed a similar theory to estimate the behaviour of a laterally-loaded vertical rigid pile in cohesive soil. The variation of the normalized ultimate lateral pile capacity H_u/c_ud^2 with μ and L/d for no moment at the ground point is shown in Fig. 21. Once again, the dependence of the ultimate lateral pile capacity on the relative stiffness factor μ can be clearly observed. The normalized ultimate lateral pile capacity increases from about 39 to 67 as μ increases from 10 to 10,000 for $L/d = 20$.

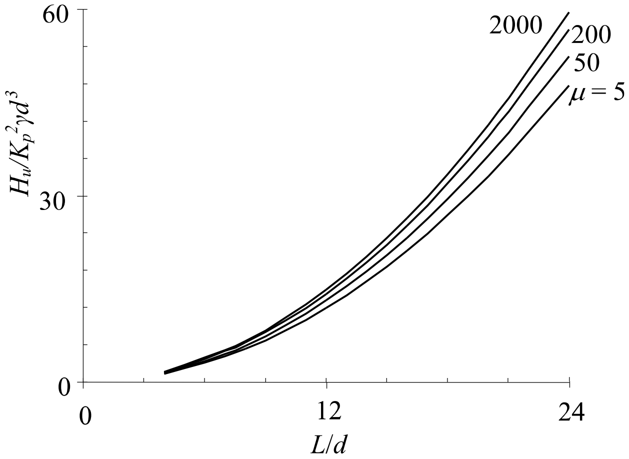

Fig. 20 $H_u/K_p^2\gamma d^3$ vs. L/d – effect of μ (pile in cohesionless soil)

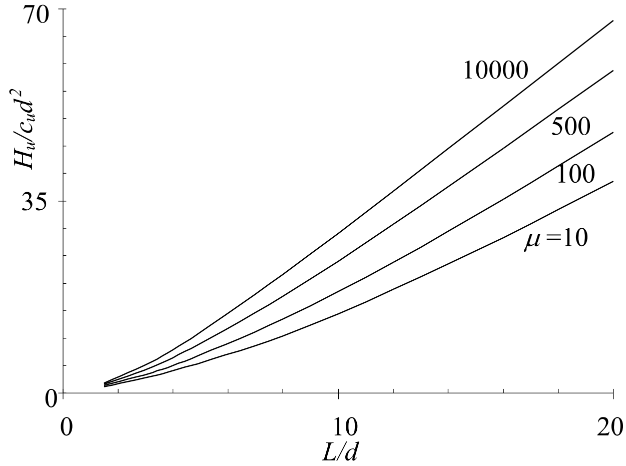

Fig. 21 H_u/c_ud^2 vs. L/d – effect of μ (pile in cohesive soil)

7. ULTIMATE CAPACITY OF TALL STRUCTURES ON GROUND (Based on Soil Stiffness Alone: Leaning Instability)

Another of Terzaghi's simplification has been the neglect of the influence of the structure on its stability. While the stability of the entire structure founded on or in the ground should have been considered, only the stability of the foundation was considered as described in the previous sections. Hambly (1985, 1990), Cheney *et al.* (1991), Lancelotta (1993) and Potts (2003) quantify the effect of the height of the structure on its stability as somewhat akin to that of buckling of long columns. Incidentally, the buckling of long slender columns is controlled by the flexural stiffness of the structure and not by the strength of the material. Fig. 22 shows a schematic of the famous Leaning Tower of Pisa while Fig. 23 shows a simple leaning instability model that consists of a tall structure of weight W resting on homogeneous ground characterized by the coefficient of subgrade reaction k_s. The center of gravity of the structure is at height h above the ground surface. The pressure under the foundation is p while the settlement is w.

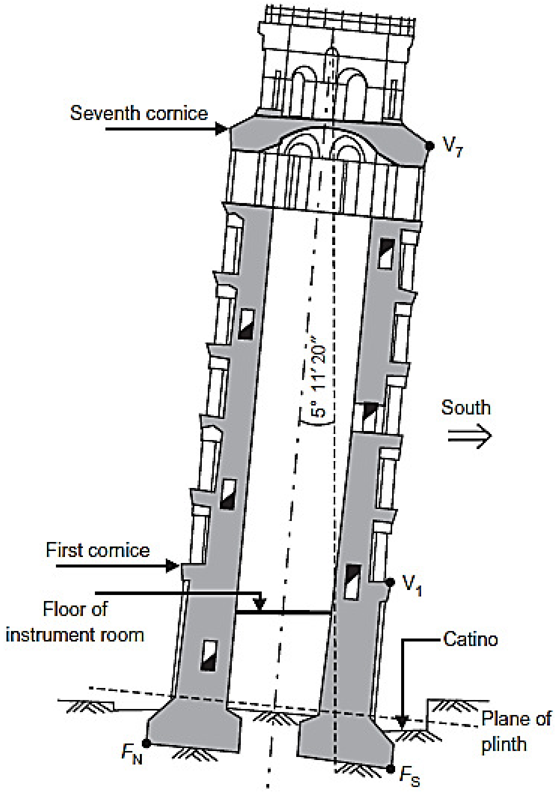

Fig. 22 Schematic of Leaning Tower of Pisa (after Potts 2003)

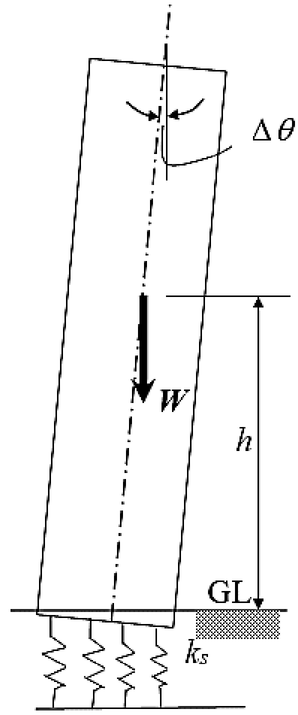

Fig. 23 Leaning instability model

If the structure leans by an angle $\Delta\theta$ due to an increase in weight ΔW, the overturning moment dM_o caused by the eccentricity of the weight about the center of the foundation (Hambly 1985), is

$$dM_0 = (W + \Delta W)h\Delta\theta \tag{18}$$

The restoring moment dM_t generated due to the variation in foundation pressure can be written as

$$dM_t = k_s I_e \Delta\theta \tag{19}$$

where I_e is the second moment of area of the foundation about the horizontal axis through the centroid. Equilibrium between the overturning and restoring moments requires that

$$(W + \Delta W)h\Delta\theta = k_s I_e \Delta\theta \tag{20}$$

At limiting equilibrium ($\Delta W = 0$), Eq. (20) can be simplified to

$$\frac{h_e w_e}{r_e^2} = 1 \tag{21}$$

where h_e is the limiting height, w_e is the average settlement, and $r_e^2 = I_e/A$, where A is the area of the foundation. Thus, the structure begins to lean when the product of the height to the center of gravity, and the average settlement w_e $(=W/Ak_s)$ equals the square of the radius of gyration r_e. If the structure is loaded non-centrally, an upper bound limit load W_y can be estimated in a similar manner, as

$$W_y = \frac{k_s I_y}{h_y} \tag{22}$$

where I_y is the second moment of area of the foundation about the horizontal axis at the edge of the foundation and h_y is the limiting height of the structure. Equations (21) and (22) demonstrate the effect of the stiffness of the ground on the leaning instability of tall structures. Lancellotta (1993) uses the concept of non-linear moment restraint to explain the phenomenon of leaning instability.

To demonstrate the leaning instability mechanism, Potts (2003) models a simple tower of 60 m height and 20 m diameter with an initial tilt of 0.5°, resting on a uniform deposit of clay with undrained shear strength s_u of 80 kPa and shear stiffness G of 10, 100 and 1000 times s_u. The stress–strain behaviour of clay was simulated using the Tresca model. According to conventional theories, the bearing capacity of the foundation of the tower was found to be the same for the three cases. In other words, if instability is governed by bearing capacity failure of the foundation, then the analyses indicate the weight of the tower to be the same for the three cases. However, if the rotation of the tower is plotted against its weight, the effect of G/s_u becomes significant (Fig. 24). The weights of the tower at failure are 60, 110 and 130 MN for G/s_u values of 10, 100 and 1000, respectively. Failure is abrupt for very stiff soils when compared to that of relatively softer soils.

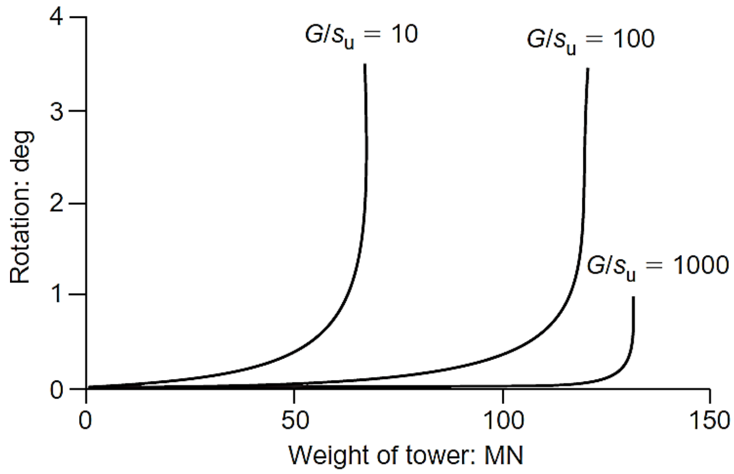

Fig. 24 Rotation of tower with increase in its weight (Potts 2003)

8. CAPACITY OF SHALLOW FOUNDATIONS FROM IN-SITU TEST PARAMETERS

(a) Based on Standard Penetration Resistance *N*

Apart from the basic soil strength and stiffness parameters often measured in the laboratory, parameters from in-situ tests have also been used to estimate the allowable bearing pressure of shallow foundations, especially in sands. Terzaghi and Peck (1967) probably gave the first empirical chart that relates the net allowable stress to the SPT blow count N and the width of the footing B (Fig. 25). The net allowable stress corresponds to a settlement w of 25 mm. These pressures were found to be conservative.

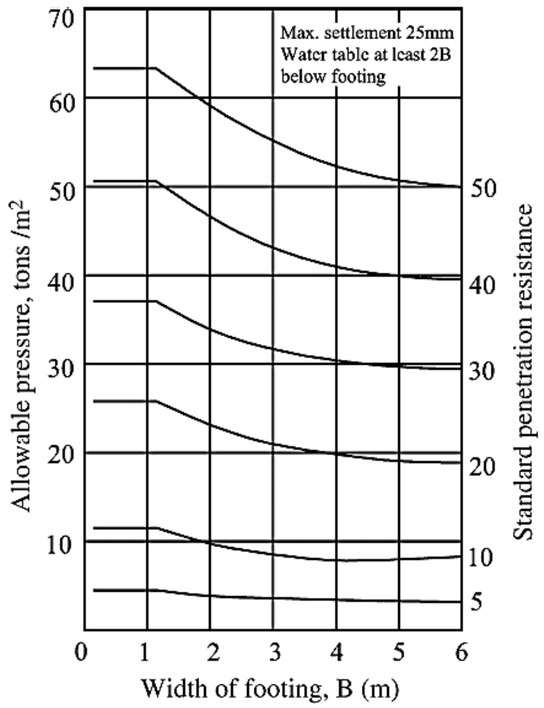

Fig. 25 Net allowable stress vs. footing width for different N values for settlement of 25 mm (Terzaghi and Peck 1967; adapted from Kameswara Rao 2011)

Meyerhof (1965) suggested a relationship for the settlement of footings in sand that is presented in terms of energy-corrected SPT blow counts and normalized quantities, as

$$\frac{w}{L_R} = \frac{0.152}{\min\left(1+\frac{D}{3B}, 1.33\right) N_{60}} \left(\frac{q_b - \sigma'_{vp}\big|_{z_f=0}}{p_A} \right) \text{; for } B \le 1.2L_R \tag{23}$$

$$\frac{w}{L_R} = \frac{0.229}{\min\left(1+\frac{D}{3B}, 1.33\right) N_{60}} \left(\frac{q_b - \sigma'_{vp}\big|_{z_f=0}}{p_A} \right) \left(\frac{B}{B+0.305L_R} \right)^2 \text{; for } B > 1.2L_R \tag{24}$$

where z_f is the depth measured from the footing base, q_b is the gross unit load at the base of the footing (including both structural loads and the weight of the backfill and foundation element), $\sigma'_{vp}\big|_{z_f=0}$ is the maximum past vertical effective stress experienced by the soil at the footing base level, N_{60} is the average SPT blow count at 60% energy ratio over a depth of 1B below the footing base for square footings and $2B$ below the footing base for strip footings, L_R is a reference length (equal to 1 m), and p_A is a reference stress (equal to 100 kPa). In Eqs. (23) and (24), the SPT N values are not corrected for water table or overburden pressure, and the min[1+D/(3B), 1.33] term is a depth factor that attempts to account for reduced settlement when the footing is embedded in the soil at a depth equal to D, all else being the same.

Based on the settlement analysis of more than 200 case records of foundations, tanks, and embankments on sands and gravels, Burland and Burbidge (1985) proposed a more reliable equation for estimating the settlement of footings, as

$$\frac{w}{L_R} = 0.1 f_s f_L f_t I_c \frac{q_b - \frac{2}{3}\sigma'_{vp}\big|_{z_f=0}}{p_A}\left(\frac{B}{L_R}\right)^{0.7} \tag{25}$$

$$f_s = \left(\frac{1.25\frac{L}{B}}{\frac{L}{B}+0.25}\right)^2 \tag{26}$$

$$f_t = \left(1 + R_3 + R_t \log\frac{t}{3}\right) \tag{27}$$

$$I_c = \frac{1.71}{\overline{N}^{1.4}} \tag{28}$$

where f_s is the shape factor, f_L is the layer thickness factor and equal to $(H/z_{f0})(2 - H/z_{f0})$ if $H \le z_{f0}$ and 1 if $H > z_{f0}$; f_t is the time factor, I_c is the compressibility index, L is the length of the footing, $\overline{N}$ is the average SPT blow count over the depth of influence z_{f0} below the footing base, H is the thickness of the sand layer, t is the time in years, R_3 is the ratio of settlement developing over a period of 3 years to the immediate settlement, and R_t is the ratio of settlement developing over a log cycle of time to the immediate settlement.

The time factor f_t captures not only the effect of increasing settlement with time but also the effect of wind loads on tall structures which can cause some additional settlement. However, in the absence of precise values for R_3 and R_t, the footing can be designed by neglecting the effect of f_t. It should be noted that $\overline{N}$ is the energy-corrected, standardized blow count N_{60}. The depth of influence z_{f0} below the footing base can be calculated from

$$\frac{z_{f0}}{L_R} = \left(\frac{B}{L_R}\right)^{0.79} \tag{29}$$

(b) Based on Cone Penetration Resistance q_c

Schmertmann (1970) and Schmertmann *et al.* (1978) proposed a reliable approach for estimation of settlement of shallow foundations in sand based on the continuous profile of q_c values obtained from static cone penetration tests. The basis for the method is that the strain in the soil mass has some value at the footing base, peaks at some depth below the footing, and then reduces to zero at the depth of influence z_{f0}. This can be observed through strain influence factor diagrams for strip ($L/B \geq 10$) and square/circular footings ($L/B = 1$) as shown in Fig. 26. The strain influence factor diagram provides values of the influence factor I_z as a function of depth z_f measured from the footing base.

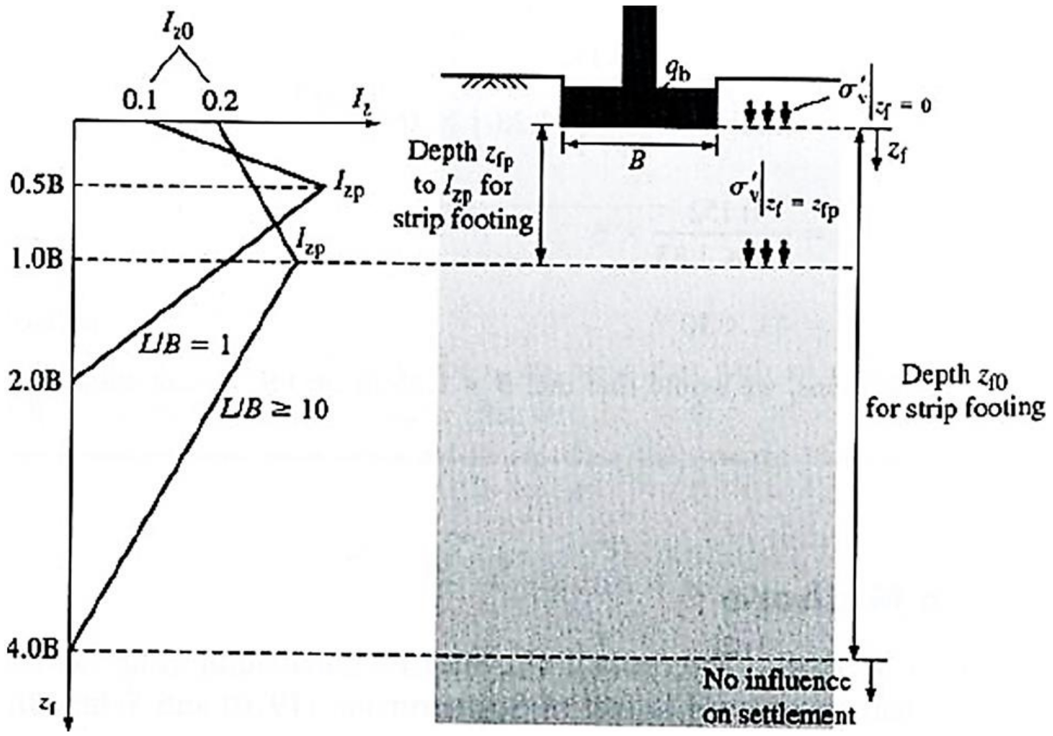

Fig. 26 Strain influence factor diagrams (Schmertmann *et al.* 1978; adapted from Salgado 2008)

After discretization of the soil profile below the footing into several sub-layers based on the q_c profile, the settlement of the footing can be computed using

$$w = C_1 C_2 \left(q_b - \sigma'_v \big|_{z_f=0} \right) \sum \left(\frac{I_{zi} \Delta z_i}{E_i} \right) \tag{30}$$

$$C_1 = 1 - 0.5 \left(\frac{\sigma'_v \big|_{z_f=0}}{q_b - \sigma'_v \big|_{z_f=0}} \right) \tag{31}$$

$$C_2 = 1 + 0.2 \log \left(\frac{t}{0.1 t_R} \right) \tag{32}$$

where C_1 and C_2 are correction factors for depth and time, respectively, $\sigma'_v \big|_{z_f=0}$ is the initial vertical effective stress at the footing base level, I_{zi} is the strain influence factor for each sub-layer, Δz_i is the thickness of each sub-layer, E_i is the representative deformation modulus of each sub-layer, t is the time, and t_R is the reference time (equal to 1 year). The depth of influence z_{f0} for rectangular footings with $1 \le L/B \le 10$ can be interpolated using the following equation

$$\frac{z_{f0}}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \le 4 \tag{33}$$

The expressions for the influence factor I_{z0} at the footing base, the depth z_{fp} below the footing base at which the influence factor peaks, and the peak influence factor I_{zp} are given by

$$I_{z0} = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \le 0.2 \tag{34}$$

$$\frac{z_{fp}}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) \le 1.0 \tag{35}$$

$$I_{zp} = 0.5 + 0.1\sqrt{\frac{q_b - \sigma'_v\big|_{z_f=0}}{\sigma'_v\big|_{z_f=z_{fp}}}} \tag{36}$$

where $\sigma'_v\big|_{z_f=z_{fp}}$ is the initial vertical effective stress at depth z_{fp} below the footing base. The influence factor I_z at any depth z_f below the footing base is given by

$$I_z = I_{z0} + \frac{z_f}{z_{fp}}\left(I_{zp} - I_{z0}\right) \text{ ; for } z_f < z_{fp} \tag{37}$$

$$I_z = \frac{z_{f0} - z_f}{z_{f0} - z_{fp}} I_{zp} \text{ ; for } z_{fp} \le z_f \le z_{f0} \tag{38}$$

The deformation modulus E_i for each sub-layer is determined based on q_c as follows: $E_i = 2.5q_c$ for young normally consolidated silica sand, $E_i = 3.5q_c$ for aged normally consolidated silica sand, and $E_i = 6.0q_c$ for overconsolidated silica sand (Schmertmann *et al.* 1978, Robertson and Campanella 1989).The methods proposed by Schmertmann *et al.* (1978) and Burland and Burbidge (1985) have been found to estimate footing settlement more reliably than the empirical methods proposed earlier by Meyerhof (1965) and Terzaghi and Peck (1967).

9. CAPACITY OF PILE FOUNDATIONS FROM IN-SITU TEST PARAMETERS

The ultimate load capacity Q_{ult} of a single pile may be expressed as the sum of the ultimate base resistance $Q_{b,ult}$ and the limit shaft resistance Q_{sL}, as

$$Q_{ult} = Q_{b,ult} + Q_{sL} = q_{b,ult}A_b + \sum_{i=1}^{n} q_{sLi}A_{si} \tag{39}$$

where $q_{b,ult}$ is the ultimate unit base resistance, q_{sLi} is the limit unit shaft resistance along the interface of the pile with soil layer i, A_b is the cross-sectional area of the pile base, A_{si} is the surface area of the pile shaft interfacing with soil layer i, and n is the number of soil layers intersected by the pile.

(a) Based on Standard Penetration Resistance *N*

The general form of the SPT-based equations for estimating $q_{b,ult}$ and q_{sLi} are

$$\frac{q_{b,ult}}{p_A} = n_b N_b \tag{40}$$

$$\frac{q_{sLi}}{p_A} = n_{si} N_{si} \tag{41}$$

where n_{si} and n_b are constants that depend on soil type and pile type, and N_b and N_{si} are representative SPT blow counts around the pile base and for soil layer i, respectively. Tables 1 and 2 present the values of n_b and n_s, respectively, for displacement and non-displacement piles in sand and clay proposed by various researchers.

Table 1 Values of n_b for piles in sand and clay

Soil type	Pile type	
	Displacement Pile	Non-Displacement Pile
Sand	(1) $n_b = 4.8$ (Aoki and Velloso 1975) (2) $n_b = 4.0$ (Meyerhof 1983)	(1) $n_b = 0.82$ (Lopes and Laprovitera 1988) (2) $n_b = 0.60$ and $q_{b,ult}/p_A \leq 45$ for drilled shafts (Reese and O'Neill 1989) (3) $n_b = 1.9$ for continuous-flight augur piles and 1.2 for drilled shafts (Neely 1991)
Clay	(1) $n_b = 0.95$ for driven piles (Aoki and Velloso 1975, Aoki *et al.* 1978)	(1) $n_b = 0.475$ for drilled shafts (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) $n_b = 0.34$ (Lopes and Laprovitera 1988)

Table 2 Values of n_s for piles in sand and clay

Soil type	Pile type	
	Displacement Pile	Non-Displacement Pile
Sand	(1) $n_s = 0.02$ (Thorburn and MacVicar 1971) (2) $n_s = 0.033$ (Aoki and Velloso 1975, Aoki *et al.* 1978) (3) $n_s = 0.02$ for full-displacement piles and 0.01 for H-piles (Meyerhof 1976, 1983)	(1) $n_s = 0.0165$ for drilled shafts (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) $n_s = 0.014$ (Lopes and Laprovitera 1988)
Clay	(1) $n_s = 0.029$ for driven piles (Aoki and Velloso 1975, Aoki *et al.* 1978)	(1) $n_s = 0.0145$ for drilled shafts (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) $n_s = 0.024$ (Lopes and Laprovitera 1988)

(b) Based on Cone Penetration Resistance q_c

The general form of the CPT-based equations for estimating $q_{b,ult}$ and q_{sLi} are

$$q_{b,ult} = c_b q_{cb} \tag{42}$$

$$q_{sLi} = c_{si} q_{ci} \tag{43}$$

where c_b and c_{si} are constants that depend on soil type and pile type, q_{cb} is the representative cone resistance at the pile base level, and q_{ci} is the representative cone resistance for soil layer i. Tables 3 and 4 present the values of c_b and c_s, respectively, for displacement and non-displacement piles in sand and clay proposed by various researchers.

Table 3 Values of c_b for piles in sand and clay

Soil type	Pile type	
	Displacement Pile	Non-Displacement Pile
Sand	(1) c_b = 0.35–0.50 (Chow 1997) (2) c_b = 0.40 (Randolph 2003) (3) c_b = 0.20–0.70 for D_R = 30–90% (Lee and Salgado 1999, Basu *et al.* 2005) (4) c_b = 0.52 – 0.4IFR for open-ended steel pipe piles (Lee *et al.* 2003) (5) c_b = 1.02 – 0.0051D_R (Foye *et al.* 2009)	(1) c_b = 0.2 for drilled shafts (Franke 1989) (2) c_b = 0.13 ± 0.02 (Ghionna *et al.* 1994) (3) c_b = 0.23×exp(–0.0066D_R) (Salgado 2006)
Clay	(1) c_b = 0.35 for driven piles in London clay and 0.30 for jacked piles in stiff clays (Price and Wardle 1982) (2) c_b = 0.9–1.0 for all piles with full cross-section in soft to lightly overconsolidated clays (Salgado 2008)	

Note: IFR = incremental filling ratio, D_R = relative density

Table 4 Values of c_s for piles in sand and clay

Soil type	Pile type	
	Displacement Pile	Non-Displacement Pile
Sand	(1) c_s = 0.004 for driven piles (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) c_s = 0.008 for open-ended steel pipe piles, 0.012 for precast concrete and closed-ended steel pipe piles and 0.018 for Franki and timber piles (Schmertmann 1978) (3) c_s = $q_{sL}/(q_c - u)$ = 0.0034–0.006 for steel and concrete piles with full cross section (Eslami and Fellenius 1997) (4) c_s = 0.004–0.009 for D_R = 0–90% for closed-ended pipe piles (Lee *et al.* 2003) (5) c_s = 0.0015–0.004 for IFR = 0–1 for open-ended pipe piles (Lee *et al.* 2003)	(1) c_s = 0.002 for drilled shafts (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) c_s = 0.0027 (Lopes and Laprovitera 1988)
Clay	(1) c_s = 0.025 (Thorburn and MacVicar 1971) (2) c_s = 0.017 for driven piles (Aoki and Velloso 1975, Aoki *et al.* 1978) (3) c_s = 0.021–0.086 for driven piles in stiff clay to soft and sensitive clay (Eslami and Fellenius 1997)	(1) c_s = 0.0085 for drilled shafts (Aoki and Velloso 1975, Aoki *et al.* 1978) (2) c_s = 0.012 (Lopes and Laprovitera 1988)

Note: u = pore pressure at the depth corresponding to the q_c value

10. CONCLUDING REMARKS

The paper examines the ultimate limit states of shallow and deep foundations and establishes that the ultimate capacities of both shallow and deep (pile) foundations are influenced by the relative stiffness of the ground. This is in stark contrast to conventional approaches which consider only the strength parameters, c and ϕ, apart from foundation dimensions and shape, to determine the ultimate capacities. Unlike metals and concrete, ground/soil is a unique material which demands a certain finite deformation in the pre-failure stage. Vesic's cavity expansion theory provides a good base for the estimation of ultimate limit loads of shallow and deep foundations in compressible ground. Consideration of the height of a tall structure and of the whole system of structure, foundation and ground, leads to leaning instability which is controlled only by the stiffness and not by the strength of the ground. The paper finally concludes with reliable methods for the estimation of carrying capacity of shallow and deep foundations based on the two most common in-situ test methods, SPT and CPT.

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